THE COMPLETE
GAUGER.

In two Parts.

Theorical and Practical:

SHEWING,

The briefeſt Ways for Gauging all manner of regular Veſſels; whether Backs (or Coolers) Tuns, Coppers and Caſk; either the whole or the Parts, by undeniable Rules of ART.

ALSO,

All manner of irregular Veſſels, the whole or the Parts, by taking a competent Number of mean Diameters.

Likewiſe TABLETS of Allowances.

By *MICHAEL DARY*, Philomath.
heretofore Practical Gauger.

LONDON:
Printed for *Robert Horne*, and *Nathanael Ponder*, at the South Entrance of the *Royal Exchange*, and at the Peacock in the *Poultry*, near *Cornhil*. 1678.

Licensed.

October 18. 1677. Roger L'Estrange.

TO

The Right Worſhipful,

George Daſhwood,
Thomas Rowney,
William Roberts,
Samuel Daſhwood,
Felix Calverd,
John Freind,
William Strong,
Edward Buckley,
} Eſquires.

FARMERS and MANAGERS

Of His Majeſty's Revenue

OF

EXCISE,

Throughout *England* and *Wales*, and Town of *Berwick* upon *Tweed*;

THIS

Complete Gauger

Is humbly Preſented by

MICHAEL DARY.

TO THE
READER.

Courteous Reader,

IT *is now about eight or nine years since* Dr. Newton *writ his Book called* the Art of Practical Gauging: *And in the year* 1670 (*as you have it in Page* 169, *at the End of the Book called* A Guide to the young Gauger) *'tis rail'd upon, and called* Intricate, Prepoſterous, In-artificial, *and moſt* prodigiouſly Erroneous.

Indeed I was ſorry to ſee that Book ſo peſtered with Preſs Faults, though they were none of mine; the Reaſon was for that, The Dr. being called away into Herefordſhire *about his Affairs, the Preſs was not Corrected. For the making of the Table of* Segments *I gave this Rule:*

6.2831853):+0.0174532925A−S:*Q(= *the Segment required. And the very ſame Rule I give thee here again in Page* 71, *notwithſtanding all their nonſenſical Railing.*

Now, ſince the year 1669 (*when the Doctor's Book came out*) *there are ſeveral Books of Gauging come forth, and all or moſt of them are furniſhed with Tables of* Segments, *becauſe they ſee 'tis ve-*

ry neceſſary in Gauging of Cask when the Liquor cuts the Heads: And notwithſtanding that that Table of Segments *was ſo much Rail'd againſt, yet they have moſt unworthily uſurpt it, and Scribled out the firſt five Figures of it, calling it their Table, craftily obſerving where the Doctor's ſixth Figure did exceed 5, there they have rais'd their fifth Figure accordingly: Reader, I pray do me ſo much Right, and thy ſelf ſo much Information, as to look into any one of their Tables, and compare it with the Doctor's Table, and thou wilt find my words to be true. What! do they ſay the Doctors Table is full of Preſs Faults? I'le ſhew them well near as many Faults in their Curt-tail'd Tables. This is ſure and certainly a proud, beggarly, ſcribling Generation; for,* 'tis a beggarly Prodigality to make themſelves fine with other folks Feathers.

Some of them (becauſe they ſcorn acknowledgment) ſay that their Table is the Segments of a Circle whoſe Area is 2.00000, *and the* Radius *ſuppoſed to be divided by Chord-lines in the* Ratio *of* 10000 *parts; If ſo, then I'm ſure their Diameter muſt be* 20000. *Now, I pray ſee* how well their Arrows Complement one the other to 20000; *or, their* Segments *Complement one the other to* 2.00000, *which from the Title of their Table they ought to do. Nay, they have tyed a* Squib *or* Cracker (viz. *a Cypher*) *to the* End of their Arrow, *but (out and alas)* the Powder is wet and will not take Fire; *therefore no Execution can be done with this* Squib.

Others

Others of them having the very same Table yea and the Squib *too, cry out at the Top of every Page,* Sybrant Hantz his Table of Segments; *and I dare presume that they never knew* Sybrant Hantz *nor his* Table *neither; for every one knows that* Sybrant Hantz *his* Table *was made to the common* Circle *whose* Area *is* 3.14159265, &c. *its Diameter being* 2; *except they mean that little* Table *which* Metius *speaks of in his* Practical Geometry.

But the prettiest Jest of all is, that the former should cry out upon the latter for Book-rifling, *and saith he hath stollen away his* Table of Segments: *But in this I do rejoice,* That when Thieves fall out then true Folk come by their Goods.

Moreover, there be others of them that put out pretty petty-Tables of Segments, *which in reality are nothing but the tenth part of the Doctor's* Table, *as you may evidently see. (I think that the Law of this Kingdom is, That the* Parson *may warrantably take Tithes of his Parishioners, but by what Rule our Gallants take Tithe of the* Parson *I cannot tell) and these also assume the same Title with the rest; to wit, the* Area *of their* Circle *is* 2 *forsooth, and the* Radius 100.

Now let our Segment-makers *come forth from behind the Curtain, and teach my Reader how to resolve this Proposition a better way than mine.*

Prop. There is a Circle whose *Area* is 2.00000 and the *Radius* is divided into 10000 equal Parts: What is the *Area* of the *Segment* belonging

ing

ing to the verſed *Sine* whoſe length is 5000 of thoſe equal Parts?

.5000000 its Arch 30.00
its Compl. 60.00

2

120.00 = A

349066000
174533

2.094396000
0.8660254 = Sine of A

1.228370600

2 = The Area of their (Circle.

2.456741200 (0.391003 = The Segment required.
188495...

571791
565487

63042
62832

210000
188495

21505

Not .50000 as they ſay.

So their whole Table is nothing elſe but a Contradiction to the Title, as you may ſee by theſe few verſed *Sines* and *Segments*, which ought to have been inſerted according to their Title; which is enough to *abuſe* all the negligent, and to *amuſe* all the diligent Readers in *England*.

V	*Seg.*	
5000	.39100	not .50000
4000	.28476	not .37353
3000	.18812	not .25232
2000	.10408	not .14238
1000	.03740	not .05204

But

But now methinks I hear my Reader begin to say to me, Truly Friend you find fault with others for amuſing *their Readers, and you are guilty of the ſame fault your ſelf, for in page* 35 *&* 36 *I am amuſed and put to a ſtand, for methinks that* Theorem *is delivered in terms too general.*

Truly I muſt confeſs it is even ſo, and it was an inconſiderate raſhneſs in me; for that Theorem *ſhould be reſtrained only to thoſe Fruſtums whoſe Baſes are ordinate* Polygons *of an even number of Sides, and indeed the laſt Clauſe imports as much;* a Plane paſſing through the oppoſite Extremities of both Baſes: *So the* Polygons *of an odd number of Sides are excluded; therefore pray expunge* Triangular and Quinqangular, *for it was not ſpied by me till the Book was Printed and the* Errata *too.*

The Explanation of the Signs *and* Symbols *uſed in this Book, I think I need not much trouble my ſelf about, they being now adays ſo familiar to every* Youngling *in the* Mathematicks; *yet that I may not be blamed for Obſcurity, I ſhall give this Explanation.*

1. $=$ *Signifies* equal to, *and is the Sign of Equation, as,* A $=$ B, *is* A *equal to* B.

2. $>$ *Signifies* greater than, *and is the Sign of Majority; as* A $>$ B, *is* A *greater than* B.

3. $<$ *Signifies* leſſer than; *and is the Sign of* Minority: *As,* A $<$ B, *is* A *leſſer than* B.

4. $+$ *Signifies* more, *and is the ſign of* Addition: *As* $+$A$+$B, *is more* A *more* B. *But ſometime*

the

the Sign of the foremost Quantity is omitted (yet understood) as, A + B = + A + B.

5. — *Signifies* Less, *and is the Sign of* Subtraction: *As,* A — B, *is* A *less* B.

6. × *Signifies* Multiplied by, *and is the Sign of* Multiplication: *As,* A × B, *is* A *multiplied by* B. *But oftentimes the Sign of* Multiplication *is omitted (yet understood) as* AB = A × B. *Furthermore, sometimes a* Binomial, Trinomial, *&c. (to avoid many Multiplications) is represented as a* Solinomial *between two* Colons *with a Line of* Conexion *over the several Quantities: As* $\overline{:AA+AB:} \times P = AAP + ABP$. *But sometimes the Line of* Conexion *is omitted (yet understood) as,* $:SS - Dd: \times P = \overline{:SS-Dd:} \times P$.

7. *The Sign of* Division *is a Line drawn level between two or more Quantities: As,* $\frac{A+B-C}{D}$. *And is to be read thus:* A *more* B *less* C, *divided by* D. *Or sometime thus:* D) A + B — C (. *And is to be read thus:* D *dividing,* A *more* B *less* C.

8. *The Sign of* Continuation *is* $\div\div$ *As,* A, B, C, D, E, $\div\div$, *shews that these Quantities are in continual Proportion.*

9. *The Sign of* Interruption *is* :: *and denotes the middle of four Proportionals interrupted: As,* A, B, :: Y, Z. *And may be read thus: As* A *is to* B, *so is* Y *to* Z.

10. ( ) *A* Parenthesis *with a Power-note in*

it

it signifies Involution: *As*, :A—B:(2) *is the Square of* A *less* B. *Or*, :A—B:(3) *is the Cube of* A *less* B, *&c.*

11. √ *A Radical Sign with an* Index *in it signifies* Evolution: *As*, $\sqrt[2]{}$:A—B: *is the square Root of* A *less* B. *Or*, $\sqrt[3]{}$: A—B: *is the Cube Root of* A *less* B, *&c. yet through an irregular Custom,* √ *is usually taken for a Square Root.*

There be some Faults crept in, that I desire thee favorably to amend, as thou shalt find them at the latter End of the Book, and thou wilt oblige him that is

Thy well meaning Friend,

Mich. Dary.

THE Complete Gauger.

THE FIRST PART.

CHAP. I.

Of the Requiſites fitting for a Gauger's Accompliſhment.

THE Buſineſs of Gauging depends upon *Arithmetick* and *Geometry*; therefore it is requiſite that the Gauger hath ſome competent skill in both.

In *Arithmetick*, he ought to know how to Add, Subtract, Multiply, and Divide, in whole Numbers; alſo it were not amiſs if he underſtood the ſame in Fractions, both vulgar and decimal.

In *Geometry*, he ought to know that a Point hath no Parts, or is an indiviſible Sign in a Magnitude, and is the Bounds of a Line. That a Line is a Magnitude only long, or is a Length without Breadth, and is the Bounds of a Superficies. That a Superficies (or Surface) is a Magnitude, having both Length and Breadth, but no Depth, and is the Bounds of a Solid. That a Solid is a Magnitude (or Body) having Length, Breadth and Depth.

Every of theſe three kinds of Magnitudes, are meaſured by ſome Magnitude that is *homogeneal* to it. A Line is meaſured by a Line, as one lineal Inch, or one

lineal Foot, *&c.* A Superficies is meaſured by a Superficies, as one ſquare Inch, or one ſquare Foot, *&c.* A Solid is meaſured by a Solid, as one Cubical Inch, or one Cubical Foot, *&c.*

And when 'tis known how many Lineal Inches, or Lineal Feet are contained in a Line; and when 'tis known how many ſquare Inches, or ſquare Feet, are contained in a Superficies; and when 'tis known how many Cubical Inches, or Cubical Feet, are contained in a Solid: Then is the Quantity or Content of either of theſe reſpective Magnitudes ſaid to be known.

Moreover, our Gauger ſhould underſtand the meaſures of liquid Capacity uſed here in *England.*

The meaſures of Capacity commonly uſed here in *England*, are of two ſorts; either Liquid or Dry. The liquid Meaſures are thoſe wherein are meaſured all kind of liquid ſubſtances; but we ſhall not meddle with any Liquids here but Beer, Ale, or Wine: Wherefore, that he may have ſome knowledg of what he is going about, he muſt have ſome *primitive* meaſure named, and it's Capacity known; that hereby he may underſtand the Names and Capacities of thoſe which are *derivative.*

Then know ye, that the Beer or Ale Gallon (now in uſe) contains 282 Cubical Inches; and the Wine-gallon contains 231 Cubical Inches: Alſo the dry Gallon (that is, the Corn Gallon) contains 272 $\frac{1}{4}$ Cubical Inches.

A Table

A Table of Beer Measure.

Nameso Beer Veſſels	Cubical Inches	Pints	Quarts	Gallons	Firkins	Kilderkins	Barrels
1 Barrel	10152	288	144	36	4	2	1
1 kilderkin	5076	144	72	18	2	1	
1 Firkin	2538	72	36	9	1		
1 Gallon	282	8	4	1			
1 Quart	70 ½	2	1				
1 Pint	35 ¼	1					
1 Cubical Inch	1						

A Table of Ale Measure.

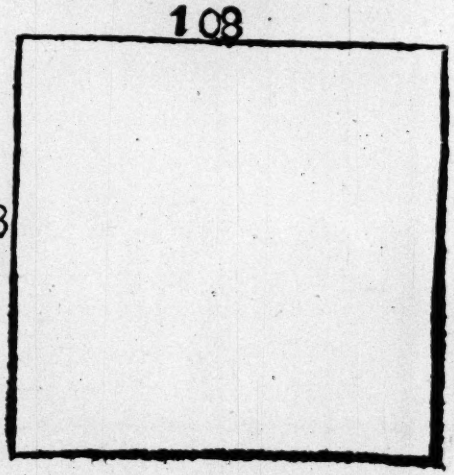
the names of Ale Vessels	Cubical Inches	Pints	Quarts	Gallons	Firkins	Kilderkins	Barrels
1 Barrel	9024	256	128	32	4	2	1
1 kilderkin	4512	128	64	16	2	1	
1 Firkin	2256	64	32	8	1		
1 Gallon	282	8	4	1			
1 Quart	70 $\frac{1}{2}$	2	1				
1 Pint	35 $\frac{1}{4}$	1					
1 Cubical Inch	1						

A Table of Wine measure.

the names of Wine Vessels	Cubical Inches	Pints	Quarts	Gallons	Runlets	Hogs-heads.	tercions	pipes or Butts.	Tuns.
1 Tun	58212	2016	1008	252	14	4	3	2	1
1 Pipe or Butt	29106	1008	504	126	7	2	1 $\frac{1}{2}$	1	
1 Tercion	19404	672	336	84	4 $\frac{2}{3}$	1 $\frac{1}{3}$	1		
1 Hogshead	14553	504	252	63	3 $\frac{1}{2}$	1			
1 Runlet	4158	144	72	18	1				
1 Gallon	231	8	4	1					
1 Quart	57 $\frac{3}{4}$	2	1						
1 Pint	28 $\frac{7}{8}$	1							
1 Cubical Inch.	1								

To find the Content of a Back or Cooler in the form of a Geometrical Square.

Rule. Multiply the Length and Breadth together, and divide the Product by 282. gives the Content in Beer, or Ale Gallons, at 1 Inch deep. If you would have the Content in Beer Barrels, divide by 10152. If in Ale Barrels, divide by 9024. *Example.*



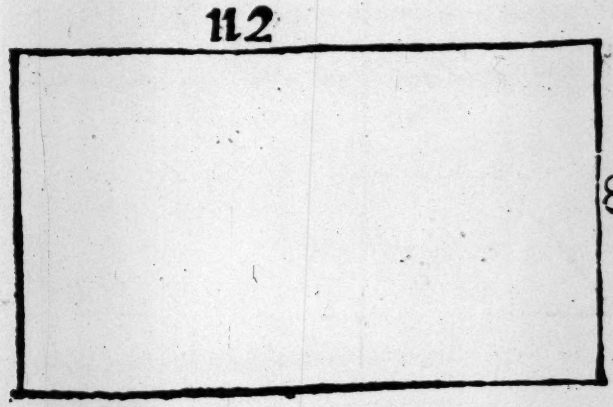
108
108
864
108 Gall.
282) 11664 (41 $\frac{102}{282}$
384
102

108
108
864
108 Beer measure.
10152) 11664 (1 : 0 : 5 $\frac{1}{2}$ *propè*
1512

108
108
864
108 Ale measure.
9024) 11664 (1 : 1 : 9 $\frac{1}{2}$ *propè*
2640

To find the Content of a Back or Cooler in the form of an Oblong.

Rule. Multiply the Length and Breadth together, and divide as beforesaid, gives the Content at 1 Inch deep. *Example.*



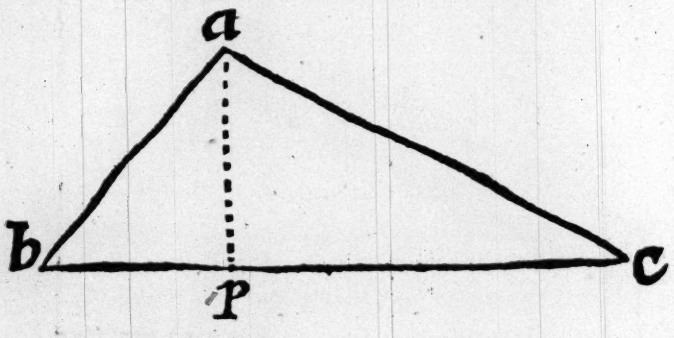
112
84
448
896 Gall.
282) 9408 (33 $\frac{1}{1+1}$
948
102

112
84
448
896 Beer measure.
10152) 9408 (0 : 3 : 6 $\frac{1}{2}$ *prope*

112
84
448
896 Ale measure.
9024) 9408 (1 : 0 : 1 $\frac{1}{2}$ *prope.*
384

To find the Content of a Back or Cooler in the form of a Triangle.

Rule. Multiply half the Baſe by the Perpendicular, or half the Perpendicular by the Baſe, and divide as beforeſaid, gives the Content at 1 Inch deep. *Example.*



$\frac{1}{2}$ *a*, *p*, = 81
b, *c*, = 241

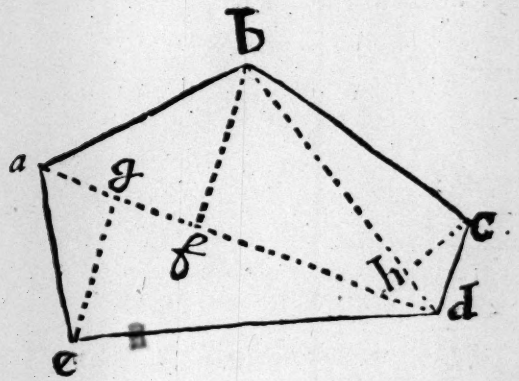
241
81
241
1928
282) 19521 (69 $\frac{213}{282}$ Gall.
2601
63

To find the Content of a Back or Cooler in the form of a Triangulate.

A Triangulate is any right-lined Figure compoſed of Triangles, and the Triangles are leſs by two than the number of Sides, and the Diagonals are leſs by three: and the Area or Content thereof is equal

to

to the Areas of all it's Triangles. *Example.*



$a, d, = 104$
$b, f, = 51$
$e, g, = 42$
$b, d, = 88$
$c, h, = 19$

Admit there were a Back or Cooler of this form; then having about me a Carpenters Line, I faſten one end at *d*, with a Nail or Aul, and then chalk the Line and ſtrain it to *b*, and ſo ſtrike out the Diagonal *d*, *b*, and keeping the Line fixt at the the Point *d*, in like manner I ſtrike out the Diagonal *d*, *a*. So having got my two Diagonals, I meaſure them with my gauging Rod, and I find *d*, *b*, = 88, and *d*, *a*, = 104.

Then to get my Perpendiculars, I faſten my chalked Line at *e*, and ſtraining of it to the Baſe *a*, *d*, I get the ſhorteſt diſtance from *e*, to *a*, *d*. which is *e*, *g*, the Perpendicular: then I do the ſame from the Point *b*, to the ſame Baſe *a*, *d*, and by this means I get the Perpendicular *b*, *f*: Laſtly, I do the ſame from the Point *c*, to the Baſe *b*, *d*, and by this means I get the

Perpendicular *c, h.* So having gotten my three Perpendiculars, I measure them, and find *e, g,* = 42 and *b, f,* = 51 and *c, h,* = 19.

e, g, = 42
b, f, = 51

93
½ *a, d,* = 52

186
465 Gall.

282)4836(17 $\frac{2}{1+1}$ The Content of the *Trapezia*,
2016 *a, b, d, e, a.*
42

c, h, = 19
½ *b, d,* = 44

76
76 Gall.

282)836(2 $\frac{1}{1+1}$ $\frac{36}{1+1}$ The Content of the Triangle,
272 *b, c, d, b.*

Gall.
So the whole Content of this Back is 20 $\frac{1}{1+1}$ $\frac{36}{1+1}$

Furthermore, that our Gauger may be farther accomplished, he must understand something in *Cyclometria*, or the quadrature of the Circle. The usual way to find the Area of a Circle, is to square it's Diameter, and multiply by 11 and divide by 14; but in regard that these numbers are too coarse when a Question requires preciseness; I shall lay down six Propositions in order to *Cyclometria.*

Proposition 1. The Diameter being given, to find the Peripheria.

Rule.

Rule. Multiply the Diameter by 3.1416, gives the Peripheria.

Prop. 2. The Diameter being given, to find the Area.

Rule. Divide the ſquare of the Diameter by 1.27324 gives the Area.

Prop. 3. The Peripheria being given, to find the Diameter.

Rule. Divide the Peripheria by 3.1416, gives the Diameter.

Prop. 4. The Peripheria being given, to find the Area

Rule. Divide the Square of the Peripheria by 12.5664, gives the Area.

Prop. 5. The Area being given, to find the Diameter.

Rule. Multiply the Area by 1.27324, and extract the Square Root gives the Diameter.

Prop. 6. The Area being given, to find the Peripheria.

Rule. Multiply the Area by 12.5664, and extract the Square Root, gives the Peripheria.

CHAP. II.

Of the Definition, and Menſuration of the three firſt kinds of Solids, to wit, a Priſm, *a* Pyramid, *and a* Pyramidoid.

NOW come we to the three firſt kinds of Solids, to wit, a *Priſm*, a *Pyramid*, and a *Pyramidoid*; upon which, chiefly, the Buſineſs of Gauging depends and

and therefore it is requiſite, that the Definition of theſe three Solids are well underſtood.

Definition 1. A *Priſm* is a *Solid* having two Baſes, equal, parallel, alike, and alike ſituate; alſo in the *Peripetaſma*, a right Line may be every where applied from Baſe to Baſe.

Under this Name *Priſm* is comprehended that round Solid uſually called a *Cylinder* by all Authors, but according to this Definition, it is indeed no more but a round *Priſm*.

Prop. 1. To find the Content of a *Priſm*, multiply the Area of one of it's Baſes by the perpendicular Height, and you have it's ſolid Content.

Definition 2. A *Pyramid* is a Solid, having one Baſe, and a Point aloft, which is called the *Vertex*; alſo in the *Peripetaſma* a right Line may be every where applied from the Baſe to the *Vertex*. Under this Name *Pyramid*, is comprehended that round Solid, uſually called by all Authors a *Cone*; but according to this Definition, it is indeed no more but a round *Pyramid*.

Prop. 2. To find the Content of a *Pyramid*, multiply the Area of it's Baſe by $\frac{1}{3}$ of the perpendicular Height, and you have it's ſolid Content.

Definition 3. A *Pyramidoid* is a *Solid* having one Baſe, and a Point aloft which is called the *Vertex*; but in the *Peripetaſma*, a right Line may be no where applied from the Baſe to the *Vertex*. Now the *Peripetaſma* of a *Pyramidoid* may be diverſe; as, Spherical, Parabolical, Hyperbolical, *&c.* But we ſhall make uſe of the two firſt only. Under the Name *Pyramidoid*, are comprehended thoſe round Solids, which are uſually by all Authors, called *Conoids*; but according to this Definition, they are no more but round *Pyramidoids*.

Prop. 3. To find the Content of a ſpherical *Pyramidoid*, multiply the Area of it's Baſe by $\frac{2}{3}$ of the Perpen-

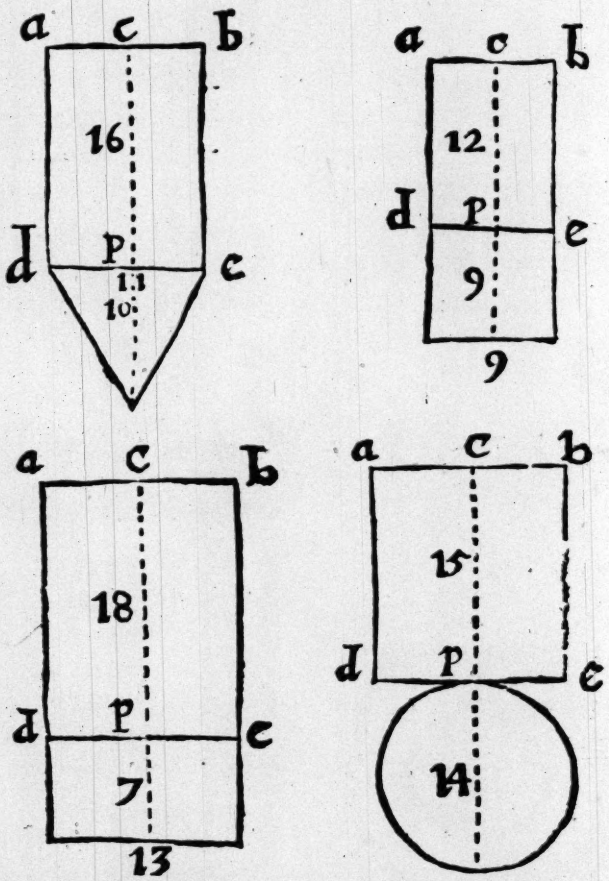
perpendicular Height, gives it's ſolid Content.

To find the Content of a parabolical *Pyramidoid*, multiply the Area of it's Baſe by ½ of the perpendicular Height, gives it's ſolid Content.

I ſhall here lay down ſome Examples of the Solids before mentioned, not according to the Art Perſpective, (for it will be hard for a Learner to apprehend them after that ſort) but according to their Orthographical Projection, letting their Baſes fall as it were upon Hinges; that ſo you may ſee the Orthographical Projection of their Baſes alſo.

Example.

Example. 1. Let there be propoſed four *Priſms*, one of a triangular Baſe, one of a ſquare Baſe, one of an oblong Baſe, and one of a circular Baſe.



For the triangular *Priſm*, the Area of it's Baſe is 55, which multiplied by C, P, = 16 the Perpendicular, is = 880 it's ſolid Content.

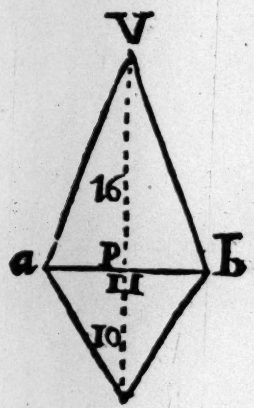
For the ſquare *Priſm* (for every *Priſm* takes it's ſur-name from it's Baſe) the Area of it's Baſe is 81, which multiplied by C,P, = 12 the Perpendicular, is = 972 it's ſolid Content.

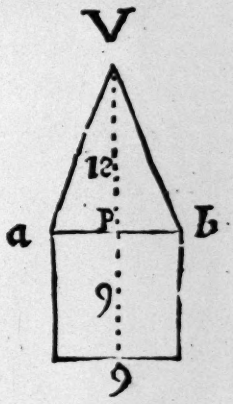
For the oblong *Priſm*, the Area of it's Baſe is 91, which

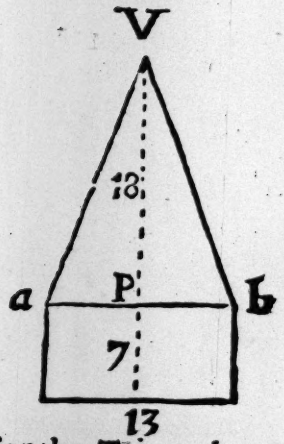
which multipled by C, P, = 18 the Perpendicular, is = 1638 it's ſolid Content.

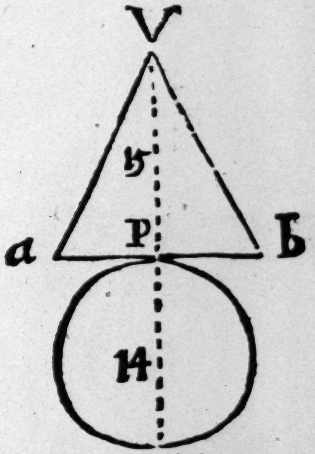
For the round *Priſm* (uſually called a *Cylinder*) the Area of it's Baſe is 153.9, which multiplied by C, P, = 15 the Perpendicular, is 2308.5 it's ſolid Content.

Example 2. Let there be propoſed four *Pyramids*, one of a triangular Baſe, one of a ſquare Baſe, one of an oblong Baſe, and one of a circular Baſe.









For the Triangular *Pyramid*, the Area of its Baſe is 55, which multiplyed by $\frac{1}{3}$ of V, P, = 5 $\frac{1}{3}$, is = 293 $\frac{1}{3}$ it's ſolid Content.

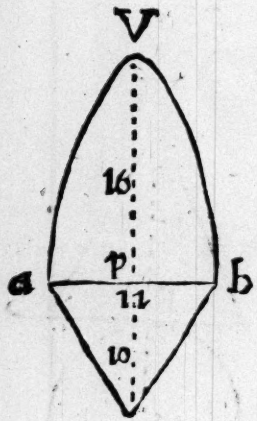
For

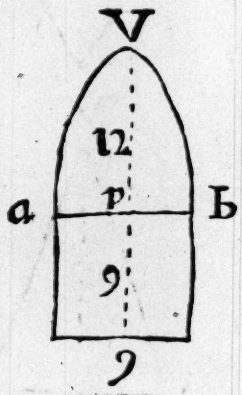
For the ſquare *Pyramid*, the Area of its Baſe is 81, which multiplied by $\frac{1}{3}$ of V,P, = 4, is = 324 it's ſolid Content.

For the oblong *Pyramid*, the Area of it's Baſe is 91, which multiplied by $\frac{1}{3}$ of V,P, = 6, is = 546 it's ſolid Content.

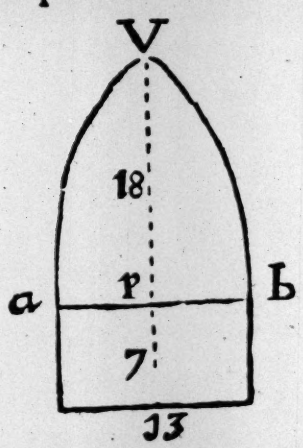
For the round *Pyramid* (uſually called a *Cone*) the Area of it's Baſe is 153.9, which multiplied by $\frac{1}{3}$ of V, P, = 5, is = 769.5 it's ſolid Content.

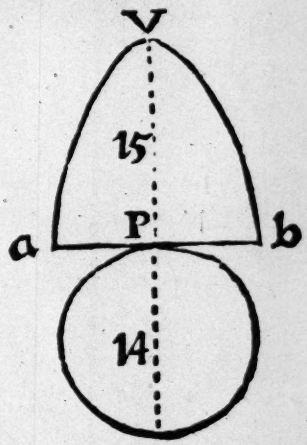
Example. 3. Let there be propoſed four *Pyramidoids*, one of a Triangular Baſe, one of a ſquare Baſe, one of an oblong Baſe, and one of a circular Baſe.





For





For the triangular *Pyramidoid*, the Area of it's Baſe is 55, which multiplied by $\frac{2}{3}$ of V, P, = 10 $\frac{2}{3}$, is = 586 $\frac{2}{3}$ its ſolid Content.

For the ſquare *Pyramidoid*, the Area of his Baſe is 81, which multiplied by $\frac{2}{3}$ of V, P, = 8, is = 648 it's ſolid Content.

For the oblong *Pyramidoid*, the Area of its Baſe is 91, which muitiplied by $\frac{2}{3}$ of V, P, = 12, is = 1092 it's ſolid Content.

For the round *Pyramidoid* (uſually called a *Conoid*) the Area of it's Baſe is 153.9, which multiplied by $\frac{2}{3}$ of V, P, = 10 is = 1539 it's ſolid Content.

CHAP. III.

Of the Definition and Menſuration of a Priſmoid.

BY the word *Priſmoid* is here to be underſtood a Solid, contained under ſix plain Surfaces, whereof the two Baſes ought to have theſe three qualifications.

1. Rectan-

1. Rectangular Parallelograms.
2. They ought to be parallel one to another.
3. A like ſituate; that is, ſo ſituate that the rectangular Conjugates in both Baſes may be inſerted by two the ſame Planes, and a right Line extended from the Center of one Baſe to the Center of the other, may be called the Axis; and the remaining four Plains are the Peripetaſma.

Under this Definition is comprehended the Fruſtum of *Pyramids* and *Priſms*, that ſhall have their Baſes rectangular Parallelograms.

Furthermore, if the Peripetaſma be not made by four flat Sides, ſpoken of before, but ſhall be conſtituted by Curvature from Circles or Ellipſes, the Solid is then called a *Cylindroid*, and under this Definition is comprehended Fruſtums of *Cones* and *Cylinders*.

Theorem. When a *Priſmoid* ſhall be proposed, if you put C = the whole Content thereof:

A & G. The two rectangular conjugates in the upper Baſe.

B & H. The two rectangular Conjugates in the neather Baſe.

A & B. Oppoſite = the two correſpondent Diameters, A in the upper Baſe, and B in the neather Baſe, inſerted by one and the ſame Plane.

G & H. Oppoſite = the two correſpondent Diameters, G in the upper Baſe, and H in the neather Baſe, alſo inſerted by one and the ſame Plane. P = the whole perpendicular Height of the *Priſmoid*.

Problem. To find C = the whole Content of a *Priſmoid* propoſed, from the five given quantities, A, B, G, H, P. The Rule thus:

: A +

$$\overline{:A+\tfrac{1}{2}B:\times G}+\overline{:B+\tfrac{1}{2}A:\times H}\times\tfrac{1}{3}P=C$$

In Words I ſhall deliver it in theſe three Precepts.

1. To that Diameter which you called A, add $\frac{1}{2}$ of that Diameter call'd B, and multiply this ſum by that Diameter called G, ſo have you attained to the firſt Member.

2. To that Diameter called B, add $\frac{1}{2}$ of that Diameter called A, and multiply this ſum by that Diameter called H, ſo have you gotten the ſecond Member.

3. Add the firſt and ſecond Members together, and multiply this ſum by $\frac{1}{3}$ of the perpendicular Height, and ſo have you obtained the ſolid Content of the *Priſmoid.* which was to be done.

But if the *Peripetaſma* (as was aforeſaid) ſhall be conſtituted by Curvature, either of Circles or Ellipſes, or both, the Solid is then a *Cylindroid:* notwithſtanding the Precepts afore going hold to all Intents and Purpoſes, only altering the laſt Clauſe of the third Precept; where it is ſaid you ſhall multiply by $\frac{1}{3}$ of the Perpendicular, you muſt in this caſe of a *Cylindroid,* multiply $\frac{1}{3.8\frac{1}{2}}$ of the Perpendicular; or which is all one, multiply firſt by the whole Perpendicular, and at laſt divide by 3.8½.

Example.

Example. There is a Tun in the form of a *Prismoid*, whose Dimensions in Inches are these:

A = 126 and G = 144
B = 102 and H = 108 } Inches.
P = 060

126 = A
51 = ½ B
177
144 = G
708
708
177
25488 = first Member.
17820 = Second Member.
43308
20 = ⅓ P.
866160 = C the solid content in cubical Inches.

102 = B
63 = ½ A
165
108 = H
1320
165
17820 = Second Member.

CHAP. IV.

Of finding the Solid Content of a Prismoid *at any perpendicular Height proposed, either more than the whole or less than the whole.*

TO perform this, you must find out three auxiliary Numbers, if there shall be so many in the *Prismoid* proposed; and these auxiliary Numbers you may call in to your Aid when you please. The Method how to find them I shall lay down in these three Rules.

Rule

Rule 1. $3\,p\,p) : \overline{A - B} : \times : \overline{G - H} : (=$ the first auxiliary Number.

Rule 2. $2\,p) : \overline{A - B} : \times H + : \overline{G - H} : \times B (=$ the second auxiliary Number.

Rule 3. $BH =$ The third auxiliary Number.

According to these three Rules, the first auxiliary Number for the Tun before going, will be $= +0.08$, the second will be $= +52.2$, and the third will be $= +11016$.

Example 1. Now let it be required to know what this Tun contains in cubical Inches, at 47 Inches wet. See the Work.

0.08 = The first auxiliary Nnmber.
47 = The Perpendicular.

56
32

3.76
52.20 = The second auxiliary Number.

55.96
47 = The Perpendicular.

39172
22384

2630.12
11016.00 = The third aux. Number.

13646.12
47 = The Perpendicular.

9552284
5458448

641367.64 = The Content in cubical Inches at 47 Inches wet.

Exam-

Example 2. There is a Tun in form of a *Prismoid*, whose Dimensions in Inches are these:

A = 129 and G = 104
B = 147 and H = 104 } Inches.
P = the Perpendicular 48

According to the three Rules before going for the finding of the auxiliary Numbers; for this Tun the first auxiliary Number is = 0, the second is = — 19.5, and the third is = + 15288. So 'tis evident that this Tun hath but two auxiliary Numbers; therefore the whole, or any part of it will be easily attained.

Let it be required to know how many cubical Inches this Tun contains, when the Perpendicular shall be 13 Inches wet. See the work.

— 19.5 = The second aux. Number.
13 = The Perpendicular.

585
195

— 253.5
+ 15288.0 = the third aux. Number.
+ 15034.5
13 = the Perpendicular.

451035
150345

195448.5 = the content at 13 Inches wet.

Example

Example 3. Let it be further required, to know how many Cubical Inches is contained in the whole Tun. See the work.

— 19.5 = the ſecond aux. Number.
48 = the Perpendicular.

1560
780

— 936.0
+ 15288.0 = The third aux. Number.

+ 14352
48 = The Perpendicular.

114816
57408

6888.96 = the whole content required.

Example 4. There is a Tun in the form of a *Priſmoid*, whoſe Dimenſions in Inches are theſe.

A = 133 and G = 205
B = 133 and H = 166 } Inches.
P = the Perpend. = 52

By the three Rules before going, for finding the auxiliary Numbers; the firſt auxiliary Number for this Tun will be = 0, the ſecond = + 49.875, and the Third = + 22078, and becauſe here will be but two auxiliary Numbers; therefore the Whole, or any part of this Tun will be eaſily had.

Let

Let it be required to know how many cubical Inches this Tun ſhall contain, when the Perpendicular is 17 wet. See the work.

49.875 = The Second aux. Number.
17 = The Perpendicular.

349125
49875
847.875
22078000 = The third aux. Number.
22925.875
17 = The Perpendicular.
160481125
22925875
389739.875 = the content in cubical Inches, at 17 Inches wet.

If it ſhall be deſir'd to give the Content of this Tun in Beer barrels, you muſt take the two auxiliary Numbers, and divide each of them by the number of cubical Inches in one Barrel; to wit, 10152, and the two Quotients ſhall exhibit to you two new auxiliary Numbers for Beer Barrels, and they will be theſe: The ſecond auxiliary Number will be = >.00492 <, and the third auxiliary Number will be = 2.17474 <. See the work.

0.00

0.00492 = The second aux. Number.
52 = The Perpendicular.

0.00984
0.2460

0.25584
2.17474 = The third aux. Number.

2.43058
52 = The Perpendicular.

4.86116
1215290

126.39016 = The whole Content in Beer Barrels.

B. F. G.
That is 126 : 1 : 4 ½ very near.

CHAP. V.

Of the Definition and Menſuration of a Cylindroid.

LEt the laſt Tun before going be ſuppoſed to be cloathed about with Curvature; that is, Let it be ſuppoſed that there is an innumerable multitude of Ellipſes inſcribed into the innumerable multitude of Oblongs that are conſtituted in the perpendicular Height of this Tun, and then it is called a *Cylindroid*. Now let it be required to find the whole Content, or any part of this *Cylindroid* in Beer Barrels. To do this, you muſt take the two auxiliary Numbers, to wit, 49 875, and 22078, and divide each of them by 12926, and the two Quotients ſhall exhibit two new auxiliary Numbers, and they ſhall be theſe; the Second auxiliary Number will be = 0.003859 >, and the Third will be = 1.708 <. See the work.

Example 1. First, let us work for the whole Content of this *Cylindroid* in Beer Barrels.

0.003859 = The second aux. Number.
52 = The Perpendicular.

0.007718
0.19295
0.200668
1.708000 = The third aux. Number.
1.908668
52
3.817336
9543340
99.250736 = The whole Content in Beer Barrels. B. F. G.
That is 99 : 1 : 0

Example 2. Furthermore, Let the Content of this *Cylindroid* in Beer Barrels be required at 20 ½ wet.

0.003859 = The second aux. Number.
20 ½ = The Perpendicular.

0.017180
19295
1.7080000 = The third aux. Number.
1.7271095
20 ½
345421900
86355475
35.4057 4475 = The Content at 20 ½ wet, in Beer Barrels.
B. F. G.
That is 35 : 1 : 6 *fere*

CHAP.

CHAP. VI.

A Compendious Method for Gauging the Fruſtum of a Pyramid.

THis way of Gauging I find in ſome of late called by the Name of *A New Way of Gauging the Fruſtum of a Pyramid*; but to me it is pretty *Old*, for I would have them to know, That I have had this way ever ſince the Year 1669, that was four or five years before ever theirs ſaw the Light.

If a Tun be an ordinate *Polygon*, the two Baſes (*i. e.* the Plane at the Top and the Plane at the Bottom) being parallel, alike, and alike ſituate; and alſo if a right Line may be every where applied in the *Peripetaſma* from Baſe to Baſe, then is that Tun the Fruſtum of a *Pyramid*, and the Content of the whole Tun or any part thereof (the Baſes being parallel to the Horizon) may be attained by this

Theorem. Let there be put A = the Side at the Top, B = the Side at the Bottom, P = the whole perpendicular Height of the Tun. Then, in the next place it will be incumbent upon you to make to your ſelf three auxiliary Numbers, by theſe three following Rules:

Rule 1. $3\,p\,p) : \overline{A - B} : (2) ($ = The firſt aux. Number.

Rule 2. $p) : \overline{A - B} : \times B ($ = The ſecond aux. Number.

Rule 3. BB = The third aux. Number.

Now that you can by theſe Rules make to your ſelf three auxiliary Numbers; *know ye*, That theſe auxiliary Numbers are only proper to the Fruſtum of a

ſquare *Pyramid*: But if you would have them fitted for any other kind of *Polygon*, they muſt each of them be divided by the Diviſor proper and peculiar for that ordinate *Polygon* to which you would have them fitted: And here in this following Tablet I have laid down the peculiar *Diviſors* for four ſorts of ordinate *Polygons*, and alſo for the *Cone*.

Diviſors proper to theſe four Polygons *and the* Cone.			
For the	Cubic inches	Ale Gallon	Beer Barrel
Trigon	2.30940	651.25	23445
Tetragon	1.00000	282.00	10152
Pentagon	0.58123	163.91	5900.6
Hexagon	0.38490	108.54	3907.5
Cone	1.27324	359.05	12926

Example 1. There is a Tun in the form of an equilateral triangular Fruſtum of a *Pyramid*; the Side at the Top is 300 Inches, the Side at the Bottom is 180 Inches, and the perpendicular Height is 30 Inches. Now let it be propoſed to find how many Beer Barrels is contained in this Tun.

By the Rules before laid down, the firſt auxiliary Number will be = + 0.000=2748 <, the ſecond will be = + 0.0307102 >, the third will be = + 1.38=96 >. See the work.

0.000

0.00022748 = The first aux. Number.
30 = The Perpendicular.

0.00682440
0.03071020 = The second aux. Number.

0.03753460
30 = The Perpendicular.

1.12603800
1.38196000 = The third aux. Number.

2.50799800
30 = The Perpendicular.

75.23994000 = The whole Content in Beer Barrels.

B. F. G.
That is 75 : 0 : 8 $\frac{1}{2}$

Example 2. Let it be required to know how many Beer Barrels is contained in this Tun, when the Perpendicular is 9 Inches wet. See the work.

0.00022748 = The first aux. Number.
9 = The Perpendicular.

0.00224732
0.03071020 = The second aux. Number.

0.03295752
9 = The Perpendicular.

0.29661780
1.38196060 = the third aux. Number.

1.67857768
9 = the Perpendicular.
15.10719912 = The Content at 9 Inches wet.

B. F. G.
That is 15 : 0 : 3 $\frac{1}{2}$.

Example 3. There is a Tun in the form of a pentagonal Frustum of a *Pyramid*, the Side at the Top

is = 144 Inches, the Side at the Bottom is = 108, and the Perpendicular is = 60 Inches.

A = 144
B = 108 } Inches.
P = 060

Let it be required to know how many Beer Barrels is contained in this Tun.

By the Rules before laid down, the first auxiliary Number for Beer Barrels will be = + 0.0000020337 >, the second will be = + 0.0109819 <, and the third will be = + 1.9767 <. See the work.

0.0000020337 = The first aux. Number.
60 = The Perpendicular.

0.001220220
0.010981900 = The second aux. Number.

0.012202120
60 = The Perpendicular.

0.732127200
1.976700000 = The third aux. Number.

2.708827200
60 = The Perpendicular.

162.529632000

B. F. G.
That is 162 : 2 : 1.

Example 4. There is a Tun in the form of a Frustum of a *Cone*, whose Dimensions are these :

Inches.
A = 144 The Diameter above.
B = 108 The Diameter below.
P = 060 The perpendicular Height.

And let it be required to find the Content in Ale Gallons.

By

By the Rules before going, the first auxiliary Number will be found to be = + 0.0003342 <, the second will be = + 0.180476 <, the third will be = + 32.4858 >. See the work.

0.0003342 = The first aux. Number.
60 = The Perpendicular.

0.0200520
0.1804760 = The second aux. Number.

0.2005280
60 = The Perpendicular.

12.0316800
32.4858000 = The third aux. Number.

44.5174800
60 = The Perpendicular.

2671.0488000 = The Content in Ale Gallons.

Before I make an end of this Chapter, I think it will be requisite to shew you how you may get the whole Content of any Frustum of a *Pyramid*, or of the Frustum of a *Cone*, without making auxiliary Numbers, by this *Theorem*.

1. Add the Side at the Top, and the Side at the Bottom together, and square this Sum, and from this Square you shall subduct the Fact (or Product) of both those Sides.

2. Multiply this Remainer by ⅓ of the Perpendicular. Lastly, divide this Product by such a Divisor as is proper and peculiar to the *Pyramid* proposed. Gives its solid Content.

Example 5. Let the Frustum of the *Cone* in the last Example be proposed, and let it be required to find its solid Content in Ale Gallons, by this *Theorem*. See the Work.

A = 144 The Diameter above.
B = 108 The Diameter below.

252
252
504
1260
504
63504
15552
47952

144
108
1152
144
15552

20 = $\frac{1}{3}$ The Perpendicular.

359.05)959040(2671.0486 = Ale Gallons.
240940
255100
37650
174500
308800
215600
170

Example 6. Let the pentagonal Tun in the third Example of this Chapter be propoſed again, and let it be propoſed to find its whole Content by this laſt *Theorem.* See the Work.

Inches
A = 144 The Side above.
B = 108 The Side below.
P = 060 The perpendicular Height.

163.91)959040.00(5851.0158 = Ale Gallons.
139490
83620
16650
25900
25090
131350
222

I might

I might here ſhew the Inching of a Fruſtum of a *Pyramid, gradatim*; by the firſt Fruſtum, firſt Difference, ſecond Difference, and third Difference; or otherwiſe, by the Table of *figurative* Numbers: But in regard you are ſufficiently inſtructed from the auxiliary numbers (in this Chapter taught) to find at any Depth propoſed, the Content upon demand; I think I may here ſpare that Labor: Howbeit, in the Second Part, when I come to ſhew Gauging by mean Diameters, I will not there leave you without it; for indeed it will be there of ſome moment, but of ſmall uſe here: for this Firſt Part preſuppoſeth every Tun to be moſt exquiſitly ſo as it is here called;, and *God wot*, you ſhall not meet with one ſo exquiſite among 5000.

CHAP. VII.

Of the Drip or Fall of a Pyramidal Tun.

EVery fixed Tun is ſo placed or poſted, that the Liquor may be deeper at the cleanſing place than at any other place; and foraſmuch as there hath been, and are yet, many great Controverſies about this Point, for want of a right underſtanding of the word *Perpendicular*; ſome taking it to be that Line which is at right Angles to the Surface of the Liquor;, others, to be that Line which is at right Angles to the Plane at the Bottom of the Tun: And, becauſe I would have a right underſtanding on all ſides, I ſhall deliver my Opinion, which I hope will clear this Point; if it doth not, I heartily wiſh that ſome abler Pen would undertake the Buſineſs, that we may have no more ſuch long Suits at Law, and Controverſies without end.

To me it ſeems reaſonable, That if the Dimenſions ſhall be taken when the Tun is dry, then is the Plane

of the Bottom the Baſe to that Tun of what form ſoever it be: Alſo, if a Point, or ſome ſignal Mark ſhall be aſſumed in the Perimeter at the Top of the Tun, for a common or conſtant Gauging place; then I do aver, that the neareſt diſtance from the aſſumed Point to the Plane at the Bottom, is the true Perpendicular to that Tun.

Objection. Some may ſay, *But there is a Tun which is the Fruſtum of a* Cone *ſtanding upon its leſſer Baſe, and conſequently, from the aſſumed Point before mentioned, it is impoſſible to get the Perpendicular to that Tun, by this way here preſcribed.*

This Objection is true, yet 'tis eaſily ſolved; for in this Caſe, I would extend a Line from the aſſumed Point, quite croſs the Perimeter at the Top of the Tun, to the oppoſite Side, and paralleſ to the Baſe or Plane of the Bottom; then would I take the neareſt diſtance from the *Croes* of the Tun, to the extended Line at the Top; and this Line I do aver to be the true Perpendicular to that Tun.

I have known ſome who have taken Dimenſions in a dry Tun (which hath been agreed upon to be the Fruſtum of a *Cone*) ſtanding with a Drip or Fall, and after Dimenſions have been taken in the Tun being dry, they have repleted the Horizon forſooth, (the lower Hoof they mean) with Liquor meaſured in by ſome known Meaſure, that ſo the quantity of Liquor which doth fill the lower Hoof might be known and determinate; and then have found by Geometry forſooth, the whole *vacant Fruſtum* above the Surface of the Liquor (even up to the aſſumed eſtabliſhed Gauging place or point) by a wrong Perpendicular; *to wit*, the Perpendicular to the Surface of the Liquor. Now this is not agreeable to Art nor Equity: For, *firſt*, There is no reaſon that they ſhould take the Diameters upon the dry Bottom, and then call them the Diameters of the Liquors Surface; for, if the Surface of the Liquor be their Baſe, then upon their Baſe they

they ought to take the Diameters, which is not at all practicable. *Secondly*, Howbeit if it were practicable, it is not the Hoof at the Bottom that is in Conteſt, but the Hoof at the Top of the Tun.

And, foraſmuch as it is apparent when a Tun ſhall have a Drip or Fall, that the vacant Hoof is at the Top of the Tun and not at the Bottom; therefore, look what quantity the Tun holds being full of Liquor, at a Horizontal Poſition; for it ſhall want ſo much of that quantity of Liquor as is equal to the vacant Hoof at the Top, the Tun ſtanding at an inclining Poſition.

Truly, I have often wondred at our able Writers of the Art of *Gauging* (who have profeſſed themſelves to be great *Hoof-Geometers*) that not one of them yet hath laid down any Rule whereby to get the vacant Hoof at the Top of a pyramidal Tun, when it ſhall have a Drip or Fall: But I'le warrant you, now they have ſeen it done here, you ſhall have them *ſcribble* it out at the next opportunity that they (have or) can pretend to expoſe themſelves in publick, and call it *their own*, as they have already done by many things which they have taken out of my Miſcellanies, which they have formerly ſo much detracted, traduced, and deſpiſed.

Notwithſtanding, I having been prevail'd upon by my loving Friend Mr. *Euclid Speidell*, a man of Honeſty, Art, and Equity; I ſhall therefore not be backward to lay down this

THEOREM.

In a Fruſtum of a Pyramid *of what form ſoever (whether Triangular, Quadrangular, Quinquangular,* &c. *yea and the* Cone *too) having its two Baſes parallel; if you put* A = *the Baſe (that is, the Area of the Baſe) at the Top,* B = *the Baſe at the Bottom, and* P = *the Perpendicular Height of the Fruſtum; Now let this Fruſtum be cut into two Hoofs, with a Plane Diagonal-wiſe, paſſing through*

through the opposite Extremities of both Bases, and then the Rule to find each Hoof is thus:

$$\overline{:A+\sqrt{\tfrac{1}{4}AB}:}\times\tfrac{1}{3}P = \text{The upper Hoof.}$$

$$\overline{:B+\sqrt{\tfrac{1}{4}AB}:}\times\tfrac{1}{3}P = \text{The neather Hoof.}$$

This Rule is universal for the Frustum of any sort of *Pyramid*, provided, its Bases are parallel: But for the Frustum of a *Cone* (which is indeed no more but a round *Pyramid*) you may attain the two Hoofs there (more easily) without knowing the Area's of its Bases, only by the knowledg of the Diameters of its Bases, and then to find each Hoof, the *Theorem* will be thus:

If you shall put A = *the Diameter (not the Area) of the upper Base,* B = *the Diameter of the neather Base, and* P = *the perpendicular Height of the Frustum; now let this Frustum be cut into two Hoofs, with a Plane Diagonal-wise, passing through the opposite Extremities of both Bases, and then the Rule to find each Hoof runs thus:*

$$3.82):\overline{AA+\tfrac{1}{2}AB}:\times P(= \text{The upper Hoof.}$$

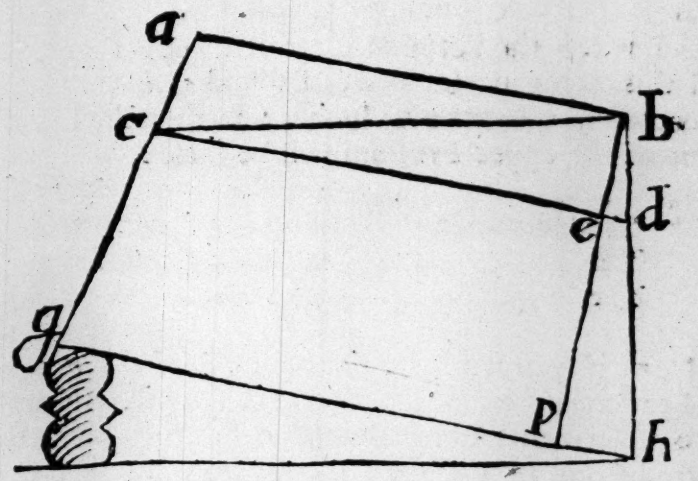
$$3.82):\overline{BB+\tfrac{1}{2}AB}:\times P(= \text{The neather Hoof.}$$

If these Dimensions were taken in Inches, then the Contents of each Hoof before going were solved in cubical Inches: But if you would have the Content of these Hoofs in Beer Barrels, you must (instead of P) multiply by $\frac{1}{2}$ P, and (instead of 3.82) divide by 12926: or if you would have the Content in Gallons Ale or Beer, you must multiply by $\frac{1}{3}$ P, and divide by 359.05.

Example.

Example. There is a Tun having a Drip or Fall, and it is agreed upon to be the Fruſtum of a *Cone*; its Dimenſions being theſe:

	Inches.
The Diameter at the Top, *a*, *b*,	= 119.24
The Diameter at the Bottom, *g*, *h*,	= 124.21
The Perpendicular, *b*, *p*,	= 042.5



Firſt, Let it be required to find the whole Content of this Tun in Beer meaſure, ſuppoſing it to ſtand in a horizontal poſition, which from the Dimenſions here given will be 53 : 1 : 2½.

Secondly, Let it be required to find the Content of the upper Hoof (*a*, *b*, *c*, *a*,) in Beer meaſure; here you muſt *note* before you can do this, you muſt get the length of the Line *c*, *d*, (parallel to *a*, *b*,) and alſo the Length of the Line *e*, *b*, which is the Perpendicular of the upper Fruſtum. In order to which, I provide me of a Line (but of the fineſt ſort) ſuch as your Carpenters uſe, and with a Pricker or an Aul I faſten one end at the Point *b*; then I extend it diametrically to the oppoſite Side, holding it very tight, and moving it downward from *a* toward *c*; and having a Partner

ner who is provided with a neat Level (such as your Masons use to try their Pavements with) applying it neatly and lighty to the tawght Line before mentioned, till it be Horizontal, which when he hath discovered, I have exactly found out the Point *c*; then I make *b*, *d*, = *a*, *c*, and with my Gauging-rod I take the Length of *c*, *d*, which is parallel to *a*, *b*, by the work: Lastly, with a pair of Compasses I take the neareſt Distance from the Point *b*, to the Line *c*, *d*, which is *e*, *b*, the Perpendicular of the upper Frustum. All this being performed, I found *c*, *d*, = 119.36 Inches, and *e*, *b*, = 2.74 Inches: So then the Dimensions of the upper Frustum will be these:

	Inches.
The Diameter at the Top, *a*, *b*, =	119.24
The Diameter at the Bottom, *c*, *d*, =	119.36
The Perpendicular---——— *e*, *b*, =	---2.74

Now, if you work for the upper Hoof (*a*, *b*, *c*, *a*,) in Beer measure, you will find it (by the Rules before prescribed) to be 1 : 2 : 0 ½, which being subducted from 53 : 1 : 2 ½ the horizontal Capacity of the whole Tun, there will remain 51 Barr. 3 Firk. 2 Gall. and so much is the just liquid quantity of this Tun, as it stands at this inclining Position. And this I will averr before any able Artist that is an unbyassed man, this Tun being taken from a true Frustum of a *Cone*, as it was agreed upon.

There be some that have decryed and derided this way of the Line and the Level, but I think they were sufficiently answered; for I pray, whether is it easier to use the Line and Level, or to cover the Bottom with (perhaps) 3 or 4 Barrels of Liquor poured in gently and cunningly? and after they have fished for the deepest place and found it, then to scoop it out again; methinks it was a great deal of toil to pour it in, and no less trouble to scoop it out again.

Yet I would have them to know, that if the lower Hoof

Hoof ſhall be juſt covered with Liquor, then I can attain thoſe Lines *c*, *d*, and *e*, *b*, at the Top, without the Line and Level; for in this very Tun here propoſed, the Liquor in the deepeſt at the cleanſing Place, did wet the Perpendicular 2.8 Inches, as ſome of them very well know; and if they pleaſe, they may ſatisfie themſelves by Calculation, whether *c*, *d*, be equal to 119.36, and *e*, *b*, equal to 2.74 or not.

If all this will not do, I wiſh there were ſome Law eſtabliſhed, that every Tun that is in conteſt might be *adjuſted*, by filling of it by ſome Standard Barrel (for it would be but a little pains more than to cover the Bottom with Liquor) and then all their prittle-prattle about *Art* (for they will not be Try'd by Art) would ceaſe, and little work left for them to do.

For I think that every man that is endued with common Capacity (that can but add 5 and 9 together, or divide 6 by 3) knows that, *If two Planes are not parallel, they cannot have one common Perpendicular*: But in all Tuns that have a Drip or Fall, the Surface of the Liquor and the Plane of the Bottom are not parallel; *Therefore* they cannot have one common (or promiſcuous) Perpendicular.

CHAP. VIII.

Of Cask-gauging, divided into eight Paragraphs.

¶ 1. OF taking (or ſuppoſing) a Cask in any of theſe four Notions. 1. As the middle Fruſtum of a *Spheroid*. 2. As the middle Fruſtum of a parabolical Spindle. 3. As the middle Fruſtum of two parabolical *Conoids*, abutting upon one common Baſe. 4. As the middle Fruſtum of two *Cones*, abutting upon one common Baſe.

If you put D = the Diameter of the Bouldge; d = the Diameter of either of the Heads, S = D + d, and

and y = D — d; L = the length of the Axis of the Veſſel, and C = the Content thereof, the Dimenſion being taken in Inches.

Prop. 1. If a Cask be taken as the middle Fruſtum of a *Spheroid*, intercepted between two Planes paralle cutting the Axis at right Angles:
Then, 3.82): 2 DD + dd: × L (= C.

Prop. 2. If a Cask be taken as the middle Fruſtum of a parabolical Spindle, intercepted between two Planes parallel, cutting the Axis at right Angles: then 3.82), 2 DD + dd — 0.4 yy: × L (= C.

Prop. 3. If a Cask be taken as the middle Fruſtum of 2 parabolical Conoids abutting upon one common Baſe, intercepted, *&c.* Then, 3.82): DD + dd: × 1 ½ L = C.

Prop. 4. If a Cask be taken as the middle Fruſtum of two Cones abutting upon one common Baſe, intercepted, *&c.* Then, 3.82): SS — Dd: × L (= C.

Now, if you would have the Content in Beer Gallons, you muſt multiply 3 82 by 282, makes your Diviſor = 1077 *prope*: If you would have Wine Gallons, 3.82 × 231 makes your Diviſor = 883 *fere*. And from theſe two Diviſors, you may Calculate Tables for Wine or Beer Meaſure: For the ſquare of any Diameter in Inches divided by 883, is the Conſtruction of the Wine Table; or by 1077, is the Conſtruction of the Beer Table: And either of theſe Tables have their ſecond Differences eqnal, therefore they will be made by an eaſie Collection *of making the Tables for Wine or Beer*.

If a Cask *be not full (the* Axis *being poſited parallel to the* Horizon*) to find the quantity of Liquor remaining in it.*

To do this, you ought to have a Table of the *Segments* of a Circle, whoſe *Area* is unity, the Diameter being divided into 1000 equal parts, and then this Approximation is the readieſt hitherto uſed, which requireth this *Data*; the whole Content of the Cask, the Diameter at the Bouldge, and the wet Portion thereof; and the Proportion runs thus:

As

As the whole Diameter, is to its wet Portion :: so is the Diameter in the Table, (*i. e.* 1000,) to its like Portion: which being sought in the versed *Sines* of the Table, abutts you to a *Segment*, by which if you multiply the whole Content of the Cask, the Product is the Content of the remaining Liquor in the Cask.

Example. A *London* Beer Barrel whose Bouldge Diameter is 23 Inches, its Head Diameter 19.9 Inches, and its Length 27.4 Inches, and let be it proposed to find its Content in Beer Gallons; being taken in each of the four Notions before specified.

1. As a Frustum of a *Spheroid, per Prop.* 1. it holds,

The doubled square of the Bouldge diameter —— = 1058
The square of the Head Diameter —— = 0396.01
The Sum —— = 1454.01
The Length —— = 27.4

581604
1017807
290802

1077)39839.874(36.991 Gall.
7529
10678
9857
1644
567

2. As

2. As a Fruſtum of a *parabolical Spindle, per Prop.* 2. it holds,

The doubled ſquare of the Bouldge Diameter	= 1058
The ſquare of the Head Diameter	= 0396.01
The Sum	= 1454.01
The ſquare of the difference of the Diameters at the Bouldge and Head multiplied by 0.4	= 3.844
Subtract, Reſts	= 1450.166
The Length	= 27.4

5800664
10151162
2900332

Gall.

1077)39734.5484(36.8937
7424
9625
10094
4018
7874
335

3. As

3. As a Fruſtum of two *parabolical Conoids, per Prop.* 3. it holds,

The ſquare of the Bouldge Diameter = 529
The ſquare of the head Diameter --- = 396.01
The ſum ——— = 925.01
The Length once and a half —— = --- 41.1

92501
92501
370004 Gall.

1077)38017.911(35.299
5707
3229
10751
10581
888

4. As a Fruſtum of two *Cones, per Prop.* 4. it holds,

The ſquare of the ſum of the Diameters at the Bouldge and Head } = 1840.41
The fact of the Diameters at the Bouldge and Head } = 457.70
Subtract Reſts = 1382.71
The Length = 27.4

553084
967897
276542 Gall.

1077)37886.254(35.177
5576
1912
8355
8164
625

¶ 2. Of

¶ 2. *Of particular things in a Cask variously given and required.*

Foraſmuch as a Cask is uſually and commonly taken for the middle Fruſtum of a *Spheroid*; let us conſider it under theſe four particulars: The Bouldge Diameter, the Head Diameter, the Length (or Axis) of the Veſſel, and the Content thereof in Beer or Ale Gallons.

In order to this Conſideration, if you put D = the Bouldge Diameter, d = the Head Diameter, L = the Length, and C = the Content of the Veſſel in Beer or Ale Gallons: Then it will follow, that if any one of theſe four particulars, D, d, L, C, be ſought, it will be diſcovered by the other three, they being given. As in theſe four Propoſitions following:

Prop. 1. Queſ. C? Data: D, d, L.

Solu. 1077) : 2 DD + dd : × L. (= C.

In words thus: To the double of the Square of the Bouldge Diameter, add the Square of the Head Diameter, and multiply this Sum by the Length of the Veſſel: Laſtly, divide this Product by 1077, and the Quotient is the Content required.

Example. There is a Cask whoſe Bouldge Diameter is = 26 Inches, its Head Diameter is = 22, and the Length is = 36. What is the Content in Ale Gallons?

The double of the ſquare of the bouldge Diameter ——— = 1352
The ſquare of the Head Diameter ——— = 484
The Sum is ——— = 1836
This Sum multiplied by the Length is — = 66096
This Product divided by 1077, the Quotient is = 61.37 Ale Gallons *prope verum*

Prop. 2.

Prop. 2. Quæſ. L? Data: D, d, C.

Solu. 2 DD + dd) 1077 C (= L.

In words thus: Multiply the Content by 1077, and divide that Product by the Sum of the double of the Square of the Bouldge Diameter, and once the Square of the Head Diameter; and the Quotient is the Length required.

Example. Let the Cask propoſed be that beforegoing, and let its Length be required.

The Content in Ale Gallons is = ---61.37
This multiplied by 1077, is---- = 66095.49
This product divided by (1836) the ſum of the double of the Square of the Bouldge Diameter, and once the Square of the Head Diameter ---- } is = 36 The length required

Prop. 3. Quæſ.: D? Data: d, L, C.

Solu. $\surd : \frac{1077\ C}{2\ L} - \frac{dd}{2} : = D.$

In words thus: Multiply the Content by 1077, and divide that Product by the Length doubled; and from this Quotient ſubtract half the Square of the Head Diameter: Laſtly, extract the Square Root of this Remainer, and you have the Bouldge Diameter ſought.

Example.

Example. Let the Cask proposed be that before-going, and let its Bouldge Diameter be required.

The Content in Ale gallons is = ----61.37
This multiplied by 1077, the Product is ——— — } = 66095.49
This Product divided by (72) the double of the length of the Vessel, the Quotient is ——— — . } = ---918
Subtract half the Square of the Head Diameter ——— } = ---242
Rests-- ——— ——— ——— = ---676
Its Square Root is——— — = -----26 The Bouldge Diameter required.

Prop. 4. Ques. d? Data: D, L, C.

Solu. $\sqrt{:\frac{1077\,C}{L} - 2\,DD} := : d.$

In words thus: Multiply the Content by 1077, and divide this Product by the Length of the Vessel, and from this Quotient subtract the double of the Square of the Bouldge Diameter: Lastly, Extract the Square Root of this Remainer, and you have the Head Diameter sought.

Example

Example. Let the Cask propoſed be that before-going, and let it be required to find its Head Diameter.

The Content in Ale Gallons is —— = 61.37
This multiplied by 1077, the product is —— = 66095.49
This Product divided by (35) the length of the Veſſel, the Quotient is —— = 1836
Subtract the double of the Square of the Bouldge —— = 1352
Reſts —— = 484
Its Square Root is —— = 22 the head Diameter required.

Moreover, if theſe four particulars, the Bouldge Diameter = D, the Head Diameter = d, the length of the Veſſel = L, and the Diagonal from the middle of the Bongue-hole to the loweſt place at either of the Heads, ſhall be put = G: Then, I ſay (let the Cask be of what form it will, ſo it be round at the Heads and Bouldge) if any one of theſe four particulars D, d, L, G, be ſought; it will be diſcovered by the other three they being given. As in theſe four Propoſitions following.

Prop. 1. Queſ. G? Data: D, d, L.

Solu. $\frac{1}{2}\sqrt{:\overline{D+d}:(2)+LL:} = G.$

In words thus: Square the Sum of the Diameters of the Bouldge and Head, and to this Square add the Square of the Length of the Veſſel, then Extract the Square Root of this Sum: Laſtly, take one half of this Square Root; So have you gotten the Diagonal which was ſought.

Example.

Solu: $\frac{1}{2}\sqrt{:D+d\times D+d+LL:} = G$

Example. There is a Cask whose Bouldge Diameter is = 26 Inches, its Head Diameter = 22 Inches, and the Length of the Vessel is = 36 Inches: What is the Length of its Diagonal?

The Square of the Sum of the Bouldge and Head Diameters is ——— } = 2304
The Square of the Length of the Cask is = 1296
The Sum is ——— = 3600
Its Square Root is ——— = 60
Its half is the Diagonal ——— = 30

Prop. 2. Quest. L? Data: D, d, G.

Solu. $\sqrt{:4GG-:\overline{D+d}:(2):}=L.$

In words thus: From the quadrupled Square of the Diagonal subtract the Square of the Sum of the Bouldge and Head Diameters: Lastly, Extract the Square Root of this Remainer: So have you gotten the Length of the Vessel.

Example. Let the Cask proposed be that beforegoing.

The quadrupled Square of the Diagonal is = 3600
The Square of the Sum of the Bouldge and Head Diameters ——— } = 2304
Subtract Rests ——— = 1296
Its Square Root is the Cask's Length ——— = 36

Prop. 3. Quest. D? Data: d, L, G.

Solu. $\sqrt{:4GG-LL:}-d=D.$

In words thus: From the quadrupled Squares of the Diagonal, subtract the Square of the Cask's length, and extract the Square Root of this Remainer; Lastly,

ly, from this Square Root ſubtract the Head-diameter; So have you gotten the Bouldge-diameter.

Example. Let the Cask propoſed be that before-going.

The quadrupled Square of the Diagonal is---	=3600
The Square of the Cask's Length is ———	=1296
Subtract Reſts ———	=2304
Its Square Root is-- ———	=---48
The Head-diameter is- ———	=---22
Subtract Reſts the Bouldge-diameter- ———	=--26

Prop. 4. Queſ. d? Data : D, L, G.

Solu. $\sqrt{:4GG-LL:}$ — D=d.

In words thus : From the quadrupled Square of the Diagonal, ſubtract the Square of the Cask's Length, and extract the Square Root of this Remainer ; laſtly, from this Square Root ſubtract the Bouldge-diameter : So have you gotten the Head-diameter.

Example. Let the Cask propoſed be that before-going.

The quadrupled Square of the Diagonal is	=3600
The Square of the Cask's Length is ———	=1296
Subtract Reſts-- ———	=2304
Its Square Root is ———	=---48
The Bouldge-diameter is ———	=---26
Subtract Reſts ———	=--22

¶ 3. *Of gauging a Cask by the Tables of* Circles Areas *in Ale or Wine Gallons.*

Example. There is a Cask whoſe Dimenſions are

 theſe;

theſe: The Bouldge-diameter = 26 Inches, the Head-diameter = 22 Inches, and the Cask's length = 36 Inches. What is the Content of this Cask in Ale Gallons?

Example.

The Bouldge-diameter 26, Tabular Number = 1.883
Repeated- page 80 ——— = 1.883
The Head-diameter 22, Tabular Number — = 1.348
The Sum — = 5.114
One-third of the Cask's Length — = 12
10228
5114
The Content in Ale-gallons — = 61.368

Let us find the Content of the ſame Cask in Wine Gallons. *Example.*

The Bouldge-diameter 26, Tabular number = 2.2984
Repeated page 76 ——— = 2.2984
The Head-diameter 22, Tabular Number- = 1.6456
The Sum — = 6.2424
One third of the Cask's Length — = 12
124848
62424
The Content in Wine Gallons — = 74.9088

¶ 4. *Of Gauging Wine Casks by way of Compendium.*

The Rule is thus. 883) : 2 DD + dd : × L (= C.

I have before (in Section the firſt of this Chapter) told you that the Diviſor for Wine-Cask is 883 too much, but 882 too little; therefore the Diviſor may be done away by this *Compendium.*

1. Tranſcribe the Dividend one Grade toward the Right Hand, and take the third part of it as it there ſtands.

2. This

2. This third part, the Number transcribed, and the Dividend, these three you shall add into one Total; then separate off four decimal Places, and your Division is effected.

Example. There is a Canary-Pipe whose Dimensions are these: the Bouldge Diameter is = 32 Inches, the Head-diameter is = 22 Inches, and the length of the Vessel is = 44 Inches: Now let it be required to find the Content in Wine Gallons.

The doubled Square of the Bouldge-diameter is = 2048
The Square of the Head-diameter is = 484
The Sum is = 2532
The Length is = 44
10128
10128
The Dividend is = 111408
The Dividend transcribed is = 111408
The third part is = 37136
The Content in Wine Gallons is = 126.2624

If you have a desire to expound the Decimal Fraction .2624 in Pints, you must multiply it by 8, and the Pints will be cast over the prime Line, and so the Answer will be 126 Gallons 2 Pints.

¶ 5. *Of Ganging a Cask by the Gauge-point.*

To do this you ought to have an Instrument to slide, and on the one Side thereof, a single Line of Numbers; and on the other Side, a double Line of Numbers.

Then by the *Golden Rule* say: As the Gauge Point (which for Wine is 17.15, but for Ale is 18.95) taken in the single Line of Numbers, is to the Length of the Cask taken in the Double Line of Numbers:

So is the mean Diameter taken in the ſingle Line of Numbers, to the Content taken in the double Line of Numbers.

Now to get a mean Diameter readily and eaſily, I think Mr. *Wingate*'s way is the nimbleſt, which is thus:

He bids you take the difference of the Bouldge and Head-diameters, and add $\frac{7}{10}$ of it to the Head-diameter, or ſubtract $\frac{3}{10}$ from the Bouldge-diameter, and either way gives you the Mean-diameter.

¶ 6. *Of Gauging a Cask by that Line which is uſually called a* Diagonal-line.

For the ready giving of the Content of a Cask either in Beer or Wine-gallons, there is a Line upon moſt *Gauging-rods*, called by the Name of a *Diagonal-line*, becauſe it is uſed as it were Diagonal-wiſe; which being put in at the middle of the Bongue-hole, down to the loweſt place at the Head; then by inſpection upon the Line at the middle of the Bongue-hole, you have the Content of the Cask either in Ale or Wine Gallons, according to the property of your *Diagonal-Line*: But you had beſt to try it at the loweſt place of each Head, and take the Arithmetical-mean, leſt the Bongue-hole ſhould not be juſt in the middle of the Cask.

¶ 7. *Of Gauging a Cask whoſe* Axis *is perpendicular to the Horizon, when Part of the Liquor is drawn out.*

Let it be ſymbolized thus: D = the Diameter at the Bouldge, d = the Diameter at the Head, a = half the Axis or Length of the Veſſel, and p = the neareſt diſtance from the Plane of the Bouldge to the Surface of the Liquor.

This Propoſition hath three Caſes, for either the Veſſel is more than half full, or leſs than half full, or or elſe it is juſt half full; but if it be juſt half full, there

there is no need of this Artifice, for it may be done by ſome of the Rules beforegoing.

Caſe 1. When the Cask is more than half full; for Ale Gallons the Rule holds thus:

$$1077) :2\,DD+dd:\times a:+3\,DDp - \frac{DD-dd}{aa}\,ppp($$

= The quantity of the remaining Liquor in Ale Gallons.

In words thus: 1. To the doubled Square of the Bouldge-diameter add the Square of the Head-diameter, and multiply this Sum by half the Length of the Veſſel: So have you gotten the firſt Member of the Dividend.

2. The triple of the Square of the Bouldge-diameter, being multiplied by the neareſt diſtance from the Plane of the Bouldge to the Surface of the Liquor is your ſecond Member.

3. The difference of the Squares of the Bouldge, and Head-diameters, being multiplied by the Cube of the neareſt Diſtance from the Plane of the Bouldge to the Surface of the Liquor, and then that Product divided by the Square of the Semi-axis, you have the third Member.

Thus having gotten the three Members of your Dividend, 'tis no great difficulty to get the quantity of the remaining Liquor in the Cask: For if you add the firſt and ſecond Members together, and from that Sum ſubtract the third Member: And laſtly, divide this Remainer by 1077, this Quotient is the quantity of the remaining Liquor in Ale Gallons, which was required.

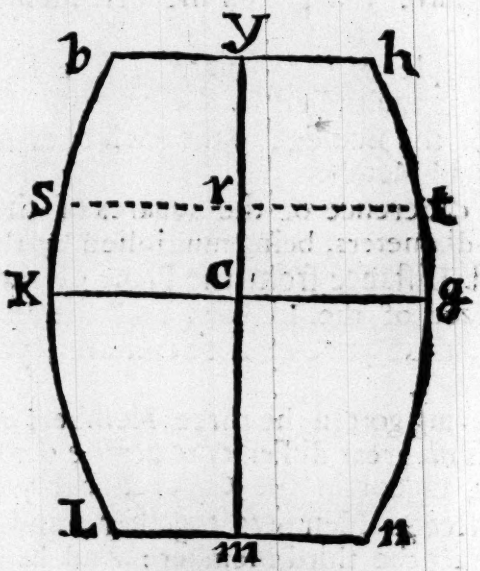
Caſe 2. When the Cask is leſs than half full: For Ale Gallons the Rule holds thus.

$$1077) : 2DD + dd : \times a - 3\,DD\,p + \frac{DD - dd}{aa} ppp ($$

= The quantity of the remaining Liquor in Ale-Gallons.

The three Members of the Dividend being gotten as before is taught, the remaining Liquor in the Cask will be had thus:

Add the first and third Members together, and from that Sum subtract the second Member: Lastly, divide this Remainer by 1077, and this Quotient is the quantity of the remaining Liquor in Ale Gallons, which was required.



K, g, = D
b, h, = d
r, c, = p
y, c, = a

Example. Let a Cask be proposed (as in the annexed Figure) whose Diameter at the Bouldge is = 26 Inches, the Diameter at the Head = 22 Inches, the

the Semi-axis = 18 Inches; and the Axis being perpendicular to the Horizon, let the Depth of the Liquor be 23 Inches. Now let it be required to know in Ale Gallons the quantity of Liquor remaining (or reſiding) in this Cask.

Here I can eaſily determine that the Cask is more than half full, in as much as the Axis is more than half wet; and it is alſo eaſie to know that p the neareſt Diſtance from the Plane of the Bouldge to the Surface of the Liquor is = 5 Inches: Now the other Dimenſions, together with p, being known, the work will appear thus:

The firſt Member is ——— = +33048
The ſecond Member is — = +10140
The third Member is ——— = ——74.074
The whole Dividend is ——— = +43113.926

This Dividend divided by 1077 is = 40.031 Ale gall.

So you ſee that this Propoſition is ſolved without producing the whole *Spheroid*; where neceſſity will require the extraction of a Square Root beſide the numerous Operations. And this Method will hold as well for Wine-gallons as Ale-gallons, if inſtead of 1077 you ſhall divide by 883.

I have been deſired by ſome *Friends* to perform this Propoſition by auxiliary Numbers, that ſo we may find the quantity of the remaining Liquor at any Height propoſed upon demand; and again on the other Hand, I have been derided and defied by ſome *buſie Prittle-prattlers* to do it that way; but to pleaſe the former and contradict the later, I will do it here.

Example 2. Let the Cask before-going be propoſed here again, and let it be required to make the three auxiliary Numbers to it. I ſay it holds,

Rule 1. $\frac{DD - dd}{-1077\,aa} =$ the first aux. Number.

Rule 2. $\frac{+3\,DD}{+1077} =$ the second aux. Number.

Rule 3. $\frac{:2DD + dd:\times a}{+1077} =$ the third aux. Number.

According to the three Rules here laid down, the first aux. Number will be found to be $=-0.00055$, the second will be $=+1.883$, and the third will be $=+30.68523$. In the last Example p was equal to 5, and we will assume it so again here. See the work.

$-0.00055 =$ The first aux. Number.
$25 = pp$
00275
00110
-0.01375
$+1.883 \cdots =$ The second aux. Number.
$+1.86925$
$5 = p$
$+9.34625$
$+30.68523 =$ The third aux. Number.
$+40.03148$ Ale Gallons.

Here you may see (notwithstanding their gain-saying) the auxiliary Numbers have performed the work exactly to that before-going.

Example 2. Let it be proposed to find the Ullage of this Cask.

Now because there is a Capacity required less than the

the half of the Veſſels Content, it falls under the ſecond Caſe; and therefore the Signs of the firſt and ſecond auxiliary Numbers are changed. See the work.

+0.00055 = The firſt aux. Number.
25 = p p
00275
00110
+0.01375
— 1.883--- = The ſecond aux. Number.
— 1.86925
5 = p
— 9.34625
+ 30.68523 = The third aux. Number.
+ 21.33898 The Ulage.

Now this Ulage and the wet part together muſt be equal to the Capacity of the whole Cask.

21.33898 the Ulage.
40.03148 the wet Part.
61.37046 the Capacity of the whole Cask.

Perhaps ſome may be inquiſitive after more than a Symbolical or Algebraical Demonſtration of theſe things, but this ſhort Tract was not intended for that Purpoſe; and they may ſatisfie themſelves out of divers Learned Authors, and not out of ſuch a curt-tail'd weak Tract as this is; yet ſome few Affections of an *Ellipſis* I will here lay down briefly.

In an Ellipſis.

1. If a right Line ſhall paſs through the Center, it is a Diameter.
2. If a Diameter ſhall biſect an Inſcript, then is that Inſcript an Ordinate.

3. If two ſeveral Diameters ſhall be cut *rato pro rato*, by two ſeveral Ordinates, then the Portions or Segments of the *Ellipſis* ſo cut off, are equal.
4. The Fact of any pair of Conjugates into the Sine of their Angle, is equal to the Fact of any other pair of Conjugates into the Sine of their Angle.
5. The ſum of the Squares of any pair of Conjugates, is equal to the ſum of the Squares of any other pair of Conjugates.
6. If the longeſt and ſhorteſt Diameters be inſcribed, they are at right Angles: And the Squares of the ſeveral Diſtances from the Center of the *Ellipſis*, of every Ordinate, are directly proportional to the differences of the Squares of every reſpective Ordinate from the Square of that Ordinate which paſſeth through the Center of the *Ellipſis*. *& contra.*
7. As the Square of the Diameter of a Circle, is to the Area of the Circle inſcribed :: So is the Parallelogram of any pair of Conjugates of an *Ellipſis*, To the Area of the *Ellipſis* inſcribed.

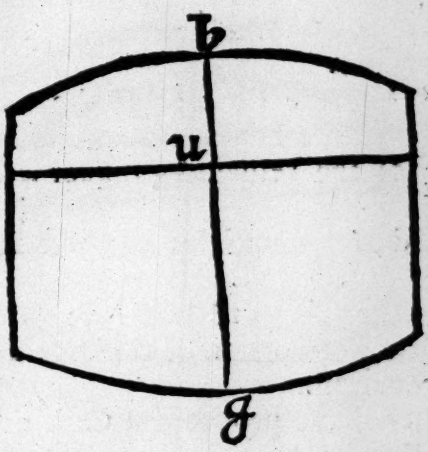
¶ 8. *Of Gauging a Cask whoſe* Axis *is poſited parallel to the Horizon, when the Liquor ſhall cut the Heads.*

Firſt, get the whole Content by ſome of the Rules beforegoing, (either Inſtrumentally or by Calculation) then you may proceed to find the quantity of the remaining Liquor in the Cask thus:

Having a Table of the *Segments* (or Portions) of a Circle whoſe Area is unity, the Diameter being cut by Chord-lines into 1000 equal parts, then this approximation is the readieſt hitherto uſed.

As the whole Diameter at the Bouldge, is to its wet Portion: So is the Diameter in the Table, (*i. e.* 1000) to its like Portion: which being ſought in the verſed Sines of the Table, abuts you to a *Segment*, by which

which if you multiply the whole Content of the Cask, the Product is the quantity of the remaining Liquor in the Cask.



Example. Let there be a Cask propoſed (as in the annexed Figure) whoſe whole Content in Ale Gallons let be 61.37, its Bouldge-diameter *b*, *g*, = 26 Inches, and its wet Portion *u*, *g*, = 17 Inches; and let it be required to find what quantity of Liquor is remaining in this Cask.

As 26 is to 17 : : So is 1000, to 0.654 *prope verum:*

This verſed Sine I ſeek in the verſed Sines to the Table of *Segments*, and it abuts me to 0.6929 a *Segment*, by which I multiply 61.37 the whole Content, and the Product is 42.523273, which is the quantity of Ale Gallons remaining in the Cask.

The Converſe of this may be propoſed thus:

Having the whole Content of a Cask, the Bouldge-diameter, and the remaining Liquor in the Cask: To find the wet portion of the Bouldge-diameter.

1. Divide the remaining Liquor in the Cask by the whole Content of the Cask, and ſeek the Quotient in the

the Table of Segments, and it abuts you to a verſed *Sine.*

2. Say by Proportion direct: As the Diameter of the Table (1000) is to the Bouldge-diameter :: So is the tabular verſed *Sine*, to the wet Portion of the Bouldge diameter.

Example. Let the Content of the Cask be 61.37 Ale Gallons, and the Bouldge-diameter 26 Inches, and the quantity of the remaining Liquor 42.523273 Ale Gallons.

The verſed *Sine* found by the firſt Precept is = 0.654.

Then, As 1000, is to 26 Inches :: So is 0.654, To 17 Inches, the wet Portion of the Bouldge-diameter, which was required.

I ſhould here have propoſed a Cask the Liquor not cutting the Heads; but that Propoſition will be ſolved by that very Rule which is given in the Second Part for the Crown of a Copper, therefore needleſs here. Howbeit you may proceed to do it moſt exquiſitly near by this 8. Paragraph, for you ſhall not miſs the $\frac{1}{1000}$ part of a Gallon, though the Liquor do not cut the Heads.

A TABLE OF *SEGMENTS*, OR PORTIONS OF A *CIRCLE*,

Whose Area is Unity.

The Diameter being cut
By Parallel Chord-lines, into
1000 equal Parts.

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg.	V	Seg.
00	.0000	100	1.0000	30	.0087	970	.9913
01	.0000	999	1.0000	31	.0092	969	.9908
02	.0001	998	.9999	32	.0096	968	.9904
03	.0003	997	.9997	33	.0101	967	.9899
04	.0004	996	.9996	34	.0105	966	.9895
05	.0006	995	.9994	35	.0110	965	.9890
06	.0008	994	.9992	36	.0115	964	.9885
07	.0010	993	.9990	37	.0120	963	.9880
08	.0012	992	.9988	38	.0124	962	.9876
09	.0014	991	.9986	39	.0129	961	.9871
10	.0017	990	.9983	40	.0134	960	.9866
11	.0020	989	.9980	41	.0139	959	.9861
12	.0022	988	.9978	42	.0144	958	.9856
13	.0025	987	.9975	43	.0149	957	.9851
14	.0028	986	.9972	44	.0154	956	.9846
15	.0031	985	.9969	45	.0160	955	.9840
16	.0034	984	.9966	46	.0165	954	.9835
17	.0037	983	.9963	47	.0171	953	.9829
18	.0041	982	.9959	48	.0176	952	.9824
19	.0044	981	.9956	49	.0181	951	.9819
20	.0048	980	.9952	50	.0187	950	.9813
21	.0051	979	.9949	51	.0193	949	.9807
22	.0055	978	.9945	52	.0198	948	.9802
23	.0059	977	.9941	53	.0204	947	.9796
24	.0063	976	.9937	54	.0210	946	.9790
25	.0067	975	.9933	55	.0215	945	.9785
26	.0070	974	.9930	56	.0221	944	.9779
27	.0075	973	.9925	57	.0227	943	.9773
28	.0079	972	.9921	58	.0233	942	.9767
29	.0083	971	.9917	59	.0239	941	.9761

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg.	V	Seg.
60	.0245	940	.9755	090	.0446	910	.9554
61	.0251	939	.9749	091	.0453	909	.9547
62	.0257	938	.9743	092	.0460	908	.9540
63	.0263	937	.9737	093	.0468	907	.9532
64	.0269	936	.9731	094	.0475	906	.9525
65	.0276	935	.9724	095	.0483	905	.9517
66	.0282	934	.9718	096	.0490	904	.9510
67	.0288	933	.9712	097	.0498	903	.9502
68	.0294	932	.9706	098	.0505	902	.9495
69	.0301	931	.9699	099	.0513	901	.9487
70	.0308	930	.9692	100	.0520	900	.9480
71	.0314	929	.9686	101	.0528	899	.9472
72	.0321	928	.9679	102	.0536	898	.9464
73	.0327	927	.9673	103	.0544	897	.9456
74	.0334	926	.9666	104	.0551	896	.9449
75	.0341	925	.9659	105	.0559	895	.9441
76	.0347	924	.9653	106	.0567	894	.9433
77	.0354	923	.9646	107	.0575	893	.9425
78	.0361	922	.9639	108	.0583	892	.9417
79	.0368	921	.9632	109	.0590	891	.9410
80	.0375	920	.9625	110	.0598	890	.9402
81	.0382	919	.9618	111	.0606	889	.9394
82	.0389	918	.9611	112	.0614	888	.9386
83	.0396	917	.9604	113	.0622	887	.9378
84	.0403	916	.9597	114	.0631	886	.9369
85	.0410	915	.9590	115	.0639	885	.9361
86	.0417	914	.9583	116	.0647	884	.9353
87	.0424	913	.9576	117	.0655	883	.9345
88	.0431	912	.9569	118	.0663	882	.9337
89	.0439	911	.9561	119	.0671	881	.9329

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg	V	Seg.
120	.0680	880	.9320	150	.0941	850	.9059
121	.0688	879	.9312	151	.0950	849	.9050
122	.0696	78	.9304	152	.0959	848	.9041
123	.0705	877	.9295	15	.0968	847	.9032
124	.0713	876	.9287	154	.0978	846	.9022
125	.0731	875	.9279	155	.0986	845	.9014
126	.0730	874	.9270	156	.0995	844	.9005
127	.0738	873	.9262	157	.1004	843	.9996
128	.0747	872	.9253	158	.1014	842	.8986
129	.0755	871	.9245	159	.1023	841	.8977
130	.0764	870	.9236	160	.1032	840	.8968
131	.0772	869	.9228	161	.1042	839	.8958
132	.0781	868	.9219	162	.1051	838	.8949
133	.0790	867	.9210	163	.1061	837	.8939
134	.0798	866	.9202	164	.1070	836	.8930
135	.0807	865	.9193	165	.1080	835	.8920
136	.0816	864	.9184	166	.1090	834	.8910
137	.0824	863	.9176	167	.1099	833	.8901
138	.0833	862	.9167	168	.1108	832	.8892
139	.0842	861	.9158	169	.1118	831	.8882
140	.0851	860	.9149	170	.1127	830	.8873
141	.0860	859	.9140	171	.1137	829	.8863
142	.0869	858	.9131	172	.1147	828	.8853
143	.0877	857	.9123	173	.1157	827	.8843
144	.0886	856	.9114	174	.1166	826	.8834
145	.0895	855	.9105	175	.1175	825	.8825
146	.0904	854	.9096	176	.1185	824	.8815
147	.0913	853	.9087	177	.1195	823	.8805
148	.0922	852	.9078	178	.1205	822	.8795
149	.0931	851	.9069	179	.1214	821	.8786

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg.	V	Seg.
180	.1224	320	.8776	210	.1526	790	.8474
181	.1234	319	.8766	211	.1537	789	.8463
182	.1243	318	.8757	212	.1547	788	.8453
183	.1253	317	.8747	213	.1558	787	.8442
184	.1263	316	.8737	214	.1568	786	.8432
185	.1273	315	.8727	215	.1578	785	.8421
186	.1283	314	.8717	216	.1589	784	.8411
187	.1293	313	.8707	217	.1599	783	.8401
188	.1303	312	.8697	218	.1610	782	.8390
189	.1313	311	.8687	219	.1620	781	.8380
190	.1323	310	.8677	220	.1631	780	.8369
191	.1333	309	.8667	221	.1642	779	.8358
192	.1343	308	.8657	222	.1652	778	.8348
193	.1353	307	.8647	223	.1663	777	.8337
194	.1363	306	.8637	224	.1673	776	.8327
195	.1373	305	.8627	225	.1684	775	.8316
196	.1383	304	.8617	226	.1695	774	.8305
197	.1393	303	.8607	227	.1705	773	.8295
198	.1403	302	.8597	228	.1716	772	.8284
199	.1413	301	.8587	229	.1727	771	.8273
200	.1424	300	.8576	230	.1737	770	.8263
201	.1434	299	.8566	231	.1748	769	.8252
202	.1444	298	.8556	232	.1759	768	.8241
203	.1454	297	.8546	233	.1770	767	.8230
204	.1465	296	.8535	234	.1780	766	.8220
205	.1475	295	.8525	235	.1791	765	.8209
206	.1485	294	.8515	236	.1802	764	.8198
207	.1495	293	.8505	237	.1813	763	.8187
208	.1506	292	.8494	238	.1824	762	.8176
209	.1516	291	.8484	239	.1834	761	.8166

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg.	V	Seg.
240	.1845	760	.8155	270	.2178	730	.7821
241	.1856	759	.8144	271	.2190	729	.7810
242	.1867	758	.8133	272	.2201	728	.7799
243	.1878	757	.8122	273	.2212	727	.7788
244	.1889	756	.8111	274	.2224	726	.7776
245	.1900	755	.8100	275	.2235	725	.7765
246	.1911	754	.8089	276	.2246	724	.7754
247	.1922	735	.8078	278	.2258	723	.7742
248	.1933	752	.8067	277	.2269	722	.7731
249	.1944	751	.8056	279	.2280	721	.7720
250	.1955	750	.8045	280	.2292	720	.7708
251	.1966	749	.8034	281	.2303	719	.7697
252	.1977	748	.8023	282	.2315	718	.7685
253	.1988	747	.8012	283	.2326	717	.7674
254	.1999	746	.8001	284	.2338	716	.7662
255	.2010	745	.7990	285	.2349	715	.7651
256	.2021	744	.7979	286	.2361	714	.7639
257	.2032	743	.7968	287	.2372	713	.7628
258	.2044	742	.7956	288	.2384	712	.7616
259	.2055	741	.7945	289	.2395	711	.7605
260	.2066	740	.7934	290	.2407	710	.7593
261	.2077	739	.7923	291	.2418	709	.7582
262	.2088	738	.7912	292	.2430	708	.7570
263	.2099	737	.7901	293	.2442	707	.7558
264	.2111	736	.7889	294	.2453	706	.7547
265	.2122	735	.7878	295	.2465	705	.7535
266	.2133	734	.7867	296	.2476	704	.7524
267	.2144	733	.7856	297	.2488	703	.7512
268	.2156	732	.7844	298	.2500	702	.7500
269	.2167	731	.7833	299	.2511	701	.7489

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg.	V	Seg.
300	.2523	700	.7477	330	.2878	670	.7122
301	.2535	699	.7465	331	.2890	669	.7110
302	.2546	698	.7454	332	.2902	668	.7098
303	.2558	697	.7442	333	.2914	667	.7086
304	.2570	696	.7430	334	.2926	666	.7074
305	.2591	685	.7419	335	.2938	665	.7062
306	.2593	694	.7407	336	.2950	664	.7050
307	.2605	693	.7395	337	.2962	663	.7038
308	.2617	692	.7383	338	.2974	662	.7026
309	.2628	691	.7372	339	.2986	661	.7014
310	.2640	690	.7360	340	.2998	660	.7002
311	.2652	689	.7348	341	.3010	659	.6990
312	.2664	688	.7336	342	.3022	658	.6978
313	.2676	687	.7324	343	.3034	657	.6966
314	.2687	686	.7313	344	.3046	656	.6954
315	.2699	685	.7301	345	.3059	655	.6942
316	.2711	684	.7289	346	.3071	654	.6929
317	.2723	683	.7277	347	.3083	653	.6917
318	.2735	682	.7265	348	.3095	652	.6905
319	.2747	681	.7253	349	.3107	651	.6893
320	.2759	680	.7241	350	.3119	650	.6881
321	.2770	679	.7230	351	.3131	649	.6869
322	.2782	678	.7218	352	.3143	648	.6857
323	.2794	677	.7206	353	.3156	647	.6844
324	.2806	676	.7194	354	.3168	646	.6832
325	.2818	675	.7182	355	.3180	645	.6820
326	.2830	674	.7170	356	.3192	644	.6808
327	.2842	673	.7158	357	.3204	643	.6796
328	.2854	672	.7146	358	.3219	642	.6784
329	.2866	671	.7134	359	.3229	641	.6771

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg.	V	Seg.
360	.3241	640	.6759	390	.3611	610	.6389
361	.3253	639	.6747	391	.3623	609	.6377
362	.3265	638	.6735	392	.3636	608	.6364
363	.3278	637	.6722	393	.3648	607	.6352
364	.3290	636	.6710	394	.3660	606	.6340
365	.3302	635	.6698	395	.3673	605	.6327
366	.3314	634	.6686	396	.3685	604	.6315
367	.3327	633	.6673	397	.3698	603	.6302
368	.3339	632	.6661	398	.3710	602	.6290
369	.3351	631	.6649	399	.3723	601	.6277
370	.3364	630	.6636	400	.3735	600	.6265
371	.3376	629	.6624	401	.3748	599	.6252
372	.3388	628	.6612	402	.3760	598	.6240
373	.3400	627	.6600	403	.3773	597	.6227
374	.3413	626	.6587	404	.3785	596	.6215
375	.3425	625	.6575	405	.3798	595	.6202
376	.3437	624	.6563	406	.3810	594	.9190
377	.3450	623	.6550	407	.3823	593	.6177
378	.3462	622	.6538	408	.3835	592	.6165
379	.3474	621	.6526	409	.3848	591	.6152
380	.3486	620	.6514	410	.3860	590	.6140
381	.3499	619	.6501	411	.3873	589	.6127
382	.3512	618	.6488	412	.3885	588	.6115
383	.3524	617	.6476	413	.3898	587	.6102
384	.3536	616	.6464	414	.3910	586	.6090
385	.3549	615	.6451	415	.3923	585	.6077
386	.3561	614	.6439	416	.3935	584	.6065
387	.3573	613	.6427	417	.3948	583	.6052
388	.3586	612	.6414	418	.3960	582	.6040
389	.3598	611	.6402	419	.3973	581	.6027

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg.	V	Seg.
420	.3986	580	.6014	450	.4364	55[illegible]	.5636
421	.3998	579	.6002	451	.4377	549	.5623
422	.4011	578	.5989	452	.4390	548	.5610
423	.4023	577	.[illegible]977	453	.4402	547	.5598
424	.4036	576	.5964	454	.4415	546	.5585
425	.4049	575	.5951	455	.4428	545	.5572
426	.4061	574	.5939	456	.4440	544	.5560
427	.4074	573	.5926	457	.4453	54[illegible]	.5547
428	.4086	572	.5914	458	.4466	542	.5534
429	.4099	571	.5901	459	.4478	541	.5522
430	.4112	570	.5888	460	.4491	540	.5509
431	.4124	[illegible]6[illegible]	.5876	461	.4504	539	.5496
432	.4137	568	.5863	462	.4516	538	.5484
433	.4149	[illegible]67	.5851	463	.4529	537	.5471
434	.4162	566	.5838	464	.4542	536	.5458
435	.4175	565	.5825	465	.4555	535	.5445
436	.4187	564	.58[illegible]3	466	.4567	534	.5433
437	.4200	563	.5800	467	.4580	533	.5420
438	.4212	562	.5788	468	.4593	532	.5407
439	.4225	561	.5775	469	.4605	531	.5395
440	.4238	560	.576[illegible]	470	.4618	530	.5382
441	.42[illegible]0	559	.575[illegible]	471	.4631	529	.5369
442	.4263	558	.5737	47[illegible]	.4644	[illegible]28	.5356
443	.4276	557	.57[illegible]4	[illegible]73	.4656	[illegible]27	.5344
444	.4288	556	.5712	474	.4[illegible]6	[illegible]26	.5331
445	.4301	555	.5699	475	.[illegible]	[illegible]25	.5318
446	.4314	554	.5686	47[illegible]	.[illegible]6[illegible]	[illegible]24	.5306
447	.4326	553	.5674	[illegible]77	.4[illegible]70	[illegible]23	.5293
448	.4339	552	.5661	478	.47[illegible]	522	.5280
449	.4352	551	.5648	[illegible]79	.[illegible]77	[illegible]21	.[illegible]267

Segments of a Circle whose Area is Unity.

V	Seg.	V	Seg.	V	Seg.	V	Seg.
480	.4743	520	.5255	490	.4867	510	.5127
481	.4758	519	.5242	491	.4885	509	.5115
482	.477	518	.5229	492	.4898	508	.5102
483	.4783	517	.5217	493	.4911	507	.5089
484	.4796	516	.5204	494	.4923	506	.5077
485	.4809	515	.5191	495	.4936	505	.5064
486	.4822	514	.5178	496	.4949	504	.5051
487	.4834	513	.5166	497	.4962	503	.5038
488	.4847	512	.5153	498	.4974	502	.5026
489	.4860	511	.5140	499	.4987	501	.5013
				500	.5000	500	.5000

CHAP. IX.

Of the Construction of the Table of Segments or Portions of a Circle.

PROBLEM.

In a Circle whose Area is Unity, and its Diameter cut by Chord-lines into 1000 *equal Parts: To find the Segment to any Verſed-Sine propoſed, not exceeding* 500 *of thoſe equal Parts.*

1. MUltiply the Verſed-Sine propoſed by 0.002, and ſubtract this Product from an Unit.

2. This Remainer you ſhall ſeek in the common Table of natural Sines (the Arch being divided into Degrees and Centeſſimals) which being found, Let its Co-Arch be doubled and called A.

3. You

3. You ſhall find the correſpondent Sine to A, which Sine being found you may call S. And then it holds.

$$6.2831853) + 0.0174532925A - S \; (= \text{Segm. required.}$$

In words thus: If you multiply 0.0174532925 by the Arch which is called A, and from this Product ſubduct away the correſpondent Sine which you called S, and then divide this Remainer by 6.2831853, the Quotient ſhall be the Segment required. Now this Segment being thus found, if you ſhall ſubduct it from an Unite, you have the Co-Segment, that is, the correſpondent Segment to the Co-verſed Sine to that verſed Sine at firſt propoſed.

But to abbreviate this large Diviſor and this large Factor, I ſhall lay down two Tabletts of them which will be ready for uſe and exact enough too.

06.2832	1
12.5664	2
18.8495	3
25.1327	4
31.4159	5
37.6991	6
43.9823	7
50.2655	8
56.5487	9

0.0174533	1
0.0349066	2
0.0523599	3
0.0698132	4
0.0872665	5
0.1047197	6
0.1221730	7
0.1396263	8
.1570796	9

Notwithſtanding what hath been ſaid in the 2d. Precept of this Problem, it very often falls out that the Remainer there ſpoken of, cannot be found in the Table of Natural Sines exactly; therefore in this Caſe my Advice is that you make two Operations, one with a Sine the next greater, and one with a Sine the next leſs; and in ſo doing you will be ſure to have the true Segment required bounded between the Reſults

ſults of theſe two Operations: Nor let any man ſay that theſe two Operations are tedious and laborious, for if he ſhould go in the common Road of ſome of our Segment-makers, and make two or three works in the Golden Rule, beſide the extraction of a Square Root which neceſſity will call for there, he would find it more tedious and laborious; and will all this do their Jobb? I trow not; for in ſpight of their Teeth (beſides all this) they muſt have the quadrature of the Circle granted them, yea and a Table of Sines to boot; and I have no more granted *me* here.

Example.

Example. Let it be proposed to find the correspondent Segment to the versed Sine 263. See the work

263 = V
.002

.526
.474 Its Arch 28.29 <
Iis Compl. 61.71 >
2

123.42 > A

349066
698132
523599
349066
174533

2.154086286
0.8346556 = Sine of A

1.319430686 (0.209993 > Seg.
125664....

627906
565487

624198
565487

587116
565487

216290
188495

27795

Now I make a second Work.

263 = V
.002
.526
.474 Its Arch 28.30 >
Its Compl. 61.70 <
2
123.40 < A
6981320
523599
349066
174533
2.153737220
0.8548478 = Sine of A
1.318889420 (0.209907 < Seg.
125664....
622494
565487
570072
565487
458500
439823
18677

So you see by these two Operations that the Segment is bounded, and 'tis very probable it may be 0.20995.

I could shew divers other ways for the making of a Table of Segments, but I will not trouble you with them at this time.

A Table

A TABLE OF *CIRCLES AREAS* IN WINE GALLONS,

And *decimilleſſimal* Parts.

To every quarter of an Inch of the DIAMETERS,

From 1 Inch to 5 Feet..

Circles Areas in Wine-Gallons.

Inches	Area	Inches	Area	Inches	Area	Inches	Area	Inches	Area
1	.0034	8	.2176	15	.7650	22	1.6456	29	2.8594
	.0053		.2314		.7907		1.6832		2.9089
	.0076		.2456		.8169		1.7213		2.9589
	.0104		.2603		.8434		1.7600		3.0093
2	.0136	9	.2754	16	.8704	23	1.7986	30	3.0600
	.0172		.2909		.8978		1.8379		3.1112
	.0212		.3069		.9257		1.8777		3.1628
	.0257		.3232		.9539		1.9180		3.2149
3	.0306	10	.3400	17	.9826	24	1.9584	31	3.2674
	.0359		.3572		1.0117		1.9994		3.3203
	.0416		.3748		1.0413		2.0409		3.3736
	.0478		.3929		1.0713		2.0827		3.4274
4	.0544	11	.4114	18	1.1016	25	2.1250	32	3.4816
	.0614		.4303		1.1324		2.1677		3.5362
	.0688		4497		1.1637		2.2108		3.5912
	.0767		.4694		1.1953		2.2544		3.6467
5	.0850	12	.4896	19	1.2274	26	2.2984	33	3.7026
	.0937		.5102		1.2599		2.3428		3.7589
	.1028		.5312		1.2928		2.3876		3.8156
	.1124		.5527		1.3262		2.4329		3.8728
6	1224	13	5746	20	1.3600	27	2.4786	34	3.9304
	.1328		.5969		1.3942		2.5247		3.9884
	.1436		.6197		1.4288		2.5712		4.0468
	.1548		.6428		1.4639		2.6181		4.1057
7	.1666	14	.6664	21	1.4994	28	2.6656	35	4.1650
	.1787		.6904		1.5353		2.7134		4.2247
	.1912		.7148		1.5717		2.7616		4.2849
	.2042		.7397		1.6084		2.8103		4.3454

Circles Areas in Wine-Gallons.

Inches	Area	Inches	Area	Inches	Area	Inches	Area	Inches	Area
36	4.4064	41	5.7154	46	7.1944	51	8.8434	56	10.6624
	4.4678		5.7853		7.2728		8.9303		10.7578
	4.5297		5.8556		7.3516		9.0176		10.8536
	4.5919		5.9264		7.4309		9.1054		10.9499
37	4.6546	42	5.9976	47	7.5106	52	9.1936	57	11.0466
	4.7177		6.0692		7.5907		9.2822		11.1437
	4.7813		6.1412		7.6713		9.3712		11.2412
	4.8452		6.2137		7.7523		9.4607		11.3392
38	4.9096	43	6.2866	48	7.8336	53	9.5506	58	11.4376
	4.9744		6.3599		7.9154		9.6409		11.5364
	5.0397		6.4336		7.9976		9.7316		11.6356
	5.1053		6.5078		8.0803		9.8228		11.7353
39	5.1714	44	6.5824	49	8.1634	54	9.9144	59	11.8354
	5.2379		6.6574		8.2469		10.0064		11.9359
	5.3049		6.7328		8.3309		10.0988		12.0368
	5.3723		6.8087		8.4152		10.1917		12.1382
40	5.4400	45	6.8850	50	8.5000	55	10.2850	60	12.2400
	5.5082		6.9617		8.5852		10.3787		
	5.5768		7.0389		8.6708		10.4728		
	5.6459		7.1164		8.7569		10.5674		

To make any Number in the precedent Table, the Rule is thus:

Multiply the Square of any Diameter by 0.0034, and the Product is the Wine-Gallons.

Example.

27
27
189
54
729
0.0034
2916
2187
2.4786

Thus you may make any Number greater or leſſer that the Table doth exhibit.

But I made this Table *gradatim*, by the continued Collection of the firſt and ſecond Differences very eaſily, as is directed in Chap. 8. Parag. 1.

A TABLE OF *CIRCLES AREAS* IN ALE GALLONS,

And Milleſſimal Parts.

TO

Every quarter of an Inch of the DIAMETERS,

From 1 Inch to 12 Feet.

Circles Areas in Ale-Gallons.

Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area
1	0.003	8	0.178	15	0.627	22	1.348	29	2.342
	0.004		0.189		0.648		1.373		2.383
	0.006		0.201		0.669		1.410		2.424
	0.008		.213		0.691		1.441		2.465
2	0.011	9	0.226	16	0.713	23	1.473	30	2.507
	0.014		0.238		0.731		1.505		2.548
	0.017		0.251		0.758		1.538		2.591
	0.021		0.265		0.781		1.571		2.634
3	0.025	10	0.278	17	0.805	24	.604	31	2.676
	0.029		0.293		0.824		1.638		2.720
	0.034		0.307		0.853		1.672		2.764
	0.039		0.322		0.879		1.706		2.808
4	0.045	11	0.337	18	0.902	25	1.741	32	2.852
	0.050		0.352		0.928		1.776		2.896
	0.056		0.368		0.953		1.811		2.942
	0.063		0.384		0.979		1.847		2.987
5	0.070	12	0.401	19	1.005	26	1.883	33	3.033
	0.077		0.418		1.027		1.919		3.079
	0.084		.435		1.059		1.956		3.126
	0.092		0.453		1.086		1.993		3.172
6	0.100	13	0.471	20	1.114	27	2.030	34	3.220
	0.109		0.489		1.142		2.068		3.267
	0.118		0.508		1.170		2.106		3.315
	0.127		0.527		1.199		2.175		3.363
7	0.136	14	0.546	21	1.228	28	2.183	35	3.412
	0.146		0.565		1.258		2.223		3.461
	0.157		0.586		1.287		2.262		3.510
	0.167		0.606		1.317		.302		3.560

Circles Areas in Ale-Gallons.

Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area
36	3.610	43	5.150	50	6.963	57	9.049
	3.666		5.210		7.033		9.128
	3.710		5.270		7.103		9.208
	3.761		5.331		7.173		9.288
37	3.813	44	5.392	51	7.244	58	9.369
	3.864		5.453		7.315		9.450
	3.916		5.515		7.387		9.531
	3.969		5.577		7.459		9.613
38	4.022	45	5.640	52	7.531	59	9.695
	4.075		5.703		7.603		9.777
	4.128		5.766		7.676		9.860
	4.182		5.829		7.750		9.943
39	4.236	46	5.893	53	7.823	60	10.029
	4.291		5.957		7.897		10.110
	4.345		6.022		7.972		10.194
	4.401		6.087		8.046		10.279
40	4.456	47	6.152	54	8.121	61	10.363
	4.512		6.218		8.197		10.448
	4.568		6.284		8.272		10.534
	4.625		6.350		8.348		10.620
41	4.682	48	6.417	55	8.425	62	10.706
	4.739		6.484		8.502		10.792
	4.797		6.551		8.579		10.879
	4.855		6.619		8.656		10.966
42	4.913	49	6.687	56	8.734	63	11.054
	4.972		6.755		8.812		11.142
	5.031		6.824		8.891		11.230
	5.090		6.893		8.970		11.319

Circles Areas in Ale-Gallons.

Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area
64	11.408	71	14.040	78	16.945	85	20.121
	11.497		14.139		17.053		20.241
	11.587		14.238		17.162		20.360
	11.677		14.338		17.272		20.479
65	11.767	72	14.438	79	17.382	86	20.599
	11.898		14.539		17.492		20.718
	11.949		14.639		17.603		20.839
	12.04-		14.740		17.714		20.959
66	12.132	73	14.842	80	17.825	87	21.080
	12.224		14.944		17.936		21.202
	12.316		15.046		18.048		21.323
	12.409		15.148		18.160		21.445
67	12.502	74	15.251	81	18.273	88	21.565
	12.596		15.354		18.386		21.691
	12.690		15.458		18.499		21.814
	12.784		15.562		18.613		21.937
68	12.878	75	15.666	82	18.727	89	22.061
	12.973		15.771		18.841		22.185
	13.068		15.876		18.956		22.309
	13.164		15.981		19.071		22.434
69	13.260	76	16.087	83	19.187	90	22.559
	13.356		16.193		19.302		22.685
	13.453		16.299		19.418		22.811
	13.550		16.406		19.535		22.937
70	13.647	77	16.513	84	19.652	91	23.063
	13.745		16.620		19.769		23.190
	13.843		16.728		19.886		23.318
	13.941		16.836		20.004		23.445

Circles Areas in Ale-Gallons.

Deam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area
92	23.573	99	27.297	106	31.293	113	35.163
	23.701		27.435		31.441		35.721
	23.830		27.573		31.589		35.878
	23.959		27.712		31.738		36.037
93	24.088	100	27.851	107	31.887	114	36.195
	24.217		27.990		32.036		36.354
	24.348		28.130		32.185		36.513
	24.478		28.270		32.335		36.673
94	24.609	101	28.411	108	32.485	115	36.833
	24.740		28.552		32.636		36.993
	24.872		28.693		32.787		37.154
	25.003		28.834		32.938		37.315
95	25.136	102	28.976	109	33.090	116	37.476
	25.269		29.118		33.242		37.638
	25.401		29.261		33.394		37.800
	25.534		29.404		33.547		37.962
96	25.667	103	29.547	110	33.700	117	38.125
	25.801		29.69		33.853		38.288
	25.935		29.835		34.007		38.452
	26.070		29.979		34.161		38.616
97	26.205	104	30.124	111	34.315	118	38.780
	26.340		30.269		34.470		38.944
	26.476		30.414		34.625		39.109
	26.611		30.560		34.780		39.274
98	26.748	105	30.706	112	34.936	119	39.440
	26.885		30.853		35.092		39.606
	27.022		30.999		35.249		39.772
	27.159		31.146		35.406		39.938

Circles

Circles Areas in Ale-Gallons.

Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area	Diam. Inches	Area
120	40.105	126	44.216	132	48.527	138	53.039
	40.273		44.392		48.712		53.232
	40.440		44.568		48.896		53.424
	40.608		44.744		49.080		53.618
121	40.777	127	44.921	133	49.266	139	53.811
	40.945		45.098		49.451		54.005
	41.114		45.275		49.637		54.199
	41.283		45.453		49.823		54.393
122	41.453	128	45.631	134	50.005	140	54.588
	41.623		45.809		50.196		54.783
	41.794		45.988		50.383		54.979
	41.965		46.167		50.571		55.174
123	42.136	129	46.347	135	50.758	141	55.371
	42.307		46.527		50.946		55.567
	42.479		46.707		51.135		55.764
	42.651		46.887		51.324		55.961
124	42.824	130	47.068	136	51.513	142	56.159
	42.997		47.249		51.703		56.357
	43.170		47.431		51.893		56.555
	43.343		47.613		52.083		56.754
125	43.517	131	47.795	137	52.273	143	56.953
	43.692		47.978		52.464		57.152
	43.866		48.161		52.656		57.352
	44.041		48.344		52.847		57.552
						144	57.752

To make any Number in the precedent Table, the Rule is thus:

Divide the Square of the Diameter by 359.05, and the Quotient is the Ale-Gallons.

Example.

Example.

30
30

359.05)900.00(2.5065
181.900
237500
220700
5270

Thus you may make any Number greater or leſſer than the Table doth exhibit, the difference being not conſiderable here, for it is not $\frac{1}{1500}$ part of a Gallon.

But I would have you underſtand that I was not ſo ignorant, as to make every Number in my Table by this Rule; for I did firſt get the Numbers to the firſt Inch, one Inch and a quarter, and one Inch and an half, and theſe to a matter of 6 places of Decimals: And then by a continued Collection of the firſt and ſecond Differences I made this Table *gradatim*, with a great deal of eaſe, as is directed in Chap. 8. Paragraph 1.

The End of the Firſt Part.

THE Complete Gauger.

THE SECOND PART.

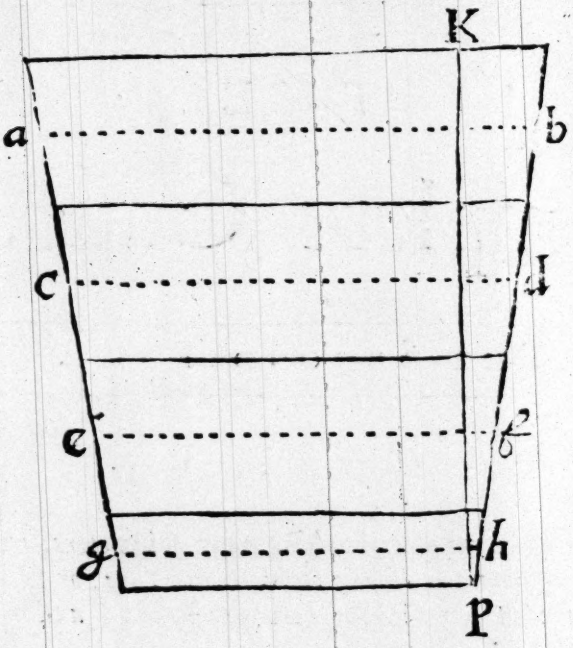
CHAP. I.

Of the uſual manner of taking mean Diameters, either in ſquare Tuns, or oblong Tuns; round Tuns, or Elliptical Tuns: And to Inch them gradatim; *and of their Drip or Fall.*

IN the Firſt Part I have ſhewed the Menſuration of Regular Solids, whereby to get their Contents; but here in this Second Part I ſhall ſhew the Menſuration of irregular Solids, whereby to get the Content of them; which is done by taking a competent Number of mean Diameters.

Now, the very taking of mean Diameters preſuppoſeth (or concludes) that the Solid is irregular in the *Peripetaſma*; for if it were regular in the *Peripetaſma*, whether it be a *Priſm*, or the Fruſtum of a *Pyramid*, *Cone*, *Conoid*, or *Spheroid*, what need have we of taking any Diameters ſave thoſe that lye in its two Baſes? So then, that which muſt be here ſpoken to, muſt be the ſhewing the uſual way of taking mean Diameters in Solids of irregular *Peripetaſmas*.

The



The uſual way of taking mean Diameters in common Brewers Tuns, is to take them in the middle of every Foot (or 12 Inches) of the Perpendicular of the Tun, but if you meaſure that Foot in the ſide Line, you muſt allow accordingly.

1. Then let this Figure here annexed repreſent a ſquare Tun, that is, every mean Diameter hath its croſs Diameter (or Conjugate) equal to its ſelf. So then here I take the firſt mean Diameter *a*, *b*, = 112 Inches; the ſecond *c*, *d*, = 109, the third *e*, *f*, = 105 ½; and the next Slice or Kerf not being a Foot (for I find it but 5 Inches thick) I take its mean Diameter (*g*, *h*, = 103 Inches) in the middle of it; that is, at 8 ½ Inches from *e*, *f*, then I place them in this order in my Book.

Perpen-

Perpendic.	Mean diameters.	Mean diameters.	Beer	Ale
12	112	- - - -	14 : 3 : 3	16 : 2 : 6
12	109	- - - -	14 : 1 : 2	16 : 0 : 3
12	105 ½	- - - -	13 : 1 : 6	15 : 0 : 3
05	103	- - - -	05 : 0 : 8	05 : 3 : 4
41			47 : 3 : 1	53 : 3 : 0

Then I ſquare the mean Diameter of every Foot, and divide that Square by 846, as in the Tablet for that purpoſe; and ſo I get the Content of every Foot in Beer meaſure.

Afterward for the loweſt Slice or Kerf, I ſquare its mean Diameter (*viz.* 103) and multiply this Square by the thickneſs of the Slice, (*viz.* 5) and divide that Product by 10152, as in the Tablet for that purpoſe; and ſo I have the Content of the loweſt Slice in Beer meaſure.

¶ *To reduce Beer meaſure to Ale meaſure,* & contra.

Beer meaſure being propoſed to be reduced to Ale meaſure, I take the 8*th.* part and ſet it under it, remembring that when I am at Barrels, if I have a Remainer, I multiply it *mentally* by 4, and add the Firkins thereto, and keep on dividing by 8, and if I have yet a Remainer, I ſet it under the Gallons; then I add all together, and I have the Ale meaſure required.

Example.

47 : 3 : 1 Beer meaſure.
5 : 3 : 7
53 : 3 : 0 Ale meaſure.

Ale

Ale meaſure being propoſed to be reduced to Beer Meaſure, I take the *9th.* part and ſet it under it, remembring that when I am at Barrels, if I have a Remainer, I multiply it *mentally* by 4, and add the Firkins thereto, and keep on dividing by 9, and if I have yet a Remainer, I ſet it under the Gallons, then I ſubtract, and the Remainer is the Beer meaſure required.

Example.

53	3	0	Ale Meaſure.
5	3	8	
47	3	1	Beer Meaſure.

2. Let the Tun beforegoing be taken for an oblong Tun, and conſequently the croſs Diameters (or Conjugates) in the middle of each Foot are not equal, and let the Dimenſions be theſe:

Perpendic.	Mean diam	Mean diam.	Beer	Ale
12	112	122	16 : 1 : 5	18 : 1 : 6
12	109	118	15 : 0 : 8	17 : 0 : 4
12	105½	115	14 : 1 : 3	16 : 0 : 4
05	103	112	05 : 2 : 6	06 : 1 : 4
41			51 : 2 : 4	58 : 0 : 2

Here I multiply together the croſs Diameters (or Conjugates) in the middle of every Foot, and divide that Product by 846, as in the Tablet for that purpoſe, and ſo I have the Content of every Foot in Beer meaſure.

Then for the loweſt Slice, I multiply its croſs Diameters (to wit, 103 and 112) together, and that Product into its thickneſs (to wit 5) and divide by 10152, as in the Tablet for that purpoſe, and ſo I have the Content of the lower Slice in Beer meaſure.

3. Let

3. Let the Tun beforegoing be taken for a round Tun, and consequently every mean Diameter hath its (cross Diameter or) Conjugate equal to it self, and Let the Dimensions be these:

Perpendic.	Mean diam.	Mean diam.	Beer	Ale
12	112	----	11 : 2 : 5	13 : 0 : 3
12	109	----	11 : 0 : 1	12 : 1 : 5
12	105½	----	10 : 1 : 3	11 : 2 : 4
05	103	----	04 : 0 : 4	04 : 2 : 4
41			37 : 0 : 4	41 : 3 : 0

Here I square the mean Diameter of every Foot, and divide that Square by 1077, as in the Tablet for that purpose, and so I get the Content of every Foot in Beer measure.

Then for the lowest Slice or Kerf, I square its mean Diameter (*viz.* 103) and multiply this Square (*viz.* 5) by the thickness of the Slice, and divide this Product by 12926, as in the Tablet for that purpose, and so I have the Content of this lowest Slice in Beer measure.

4. Let the Tun beforegoing be taken for an Elliptical Tun, and consequently the (cross Diameters or) Conjugates in the middle of each Foot are not equal; and let the Dimensions be these:

Perpendic.	Mean diam.	Mean diam.	Beer	Ale
12	112	122	12 : 2 : 7	14 : 1 : 1
12	109	118	11 : 3 : 7	13 : 1 : 6
12	105½	115	11 : 1 : 0	12 : 2 : 5
05	103	112	04 : 1 : 7	05 : 0 : 0
41			40 : 1 : 3	45 : 1 : 4

Here

Here I multiply together the (croſs Diameters or) Conjugates of every Foot, and divide that Product by 1077, as in the Tablet for that purpoſe; and ſo I obtain the Content of every Foot in Beer meaſure.

Then for the loweſt Kerf or Slice, I multiply its Conjugates (to wit, 103 and 112) together, and that Product by the thickneſs (*viz.* 5) of the Slice; then I divide this laſt Product by 12926, as in the Table for that purpoſe; and ſo I have the Content of this loweſt Slice in Beer meaſure.

Perpend.	Content Beer meaſure
0	40:1:3
1	39:1:0 $\frac{1}{12}$ $\frac{1}{2}$
2	38:0:7 $\frac{1}{12}$ $\frac{0}{2}$
3	37:0:5 $\frac{9}{12}$
4	36:0:3 $\frac{8}{12}$
5	35:0:1 $\frac{7}{12}$
6	33:3:8 $\frac{6}{12}$
7	32:3:6 $\frac{5}{12}$
8	31:3:4 $\frac{4}{12}$
9	30:3:2 $\frac{3}{12}$
10	29:3:0 $\frac{2}{12}$
11	28:2:7 $\frac{1}{12}$
12	27:2:5 0
13	26:2:5 $\frac{1}{6}$
14	25:2:5 $\frac{2}{6}$
15	24:2:5 $\frac{3}{6}$
16	23:2:5 $\frac{4}{6}$
17	22:2:5 $\frac{5}{6}$

¶ *To Inch any Tun* gradatim.

Example.

1. Under the Title *Perpendicular*, in that Column I write down the whole Perpendicular, gradually in Inches, as you ſee here done, 0, 1, 2, 3, 4, 5, *&c.*

2. Under the Title *Content Beer meaſure*, and juſt againſt 0, I write the whole Content of the Tun in Beer meaſure, as in this Example; where I have put for Illuſtration the Elliptical Tun laſt beforegoing,

3. Beginning at the Top with the firſt Foot, by the Rule of Proportion direct I ſay; If 12 Inches require 12 : 2 : 7 :: what ſhall 1 Inch require? *facit* 1 : 0 : 2 $\frac{1}{12}$: By this means I know what this firſt Foot bears upon the Inch; and if I would make this a common Subducend, I might (by continual Subduction from the whole Content) wear out the firſt Foot: but becauſe Addition is eaſier and more familiar (eſpecially when Fractions ſhall occur) I will

Perpend.	Content Beer measure
18	21:2:6 0
19	20:2:6 $\frac{1}{6}$
20	19:2:6 $\frac{2}{6}$
21	18:2:6 $\frac{3}{6}$
22	17:2:6 $\frac{4}{6}$
23	16:2:6 $\frac{5}{6}$
24	15:2:7 0
25	14:3:0 $\frac{1}{4}$
26	13:3:2 $\frac{2}{4}$
27	12:3:4 $\frac{3}{4}$
28	11:3:7 0
29	11:0:0 $\frac{1}{4}$
30	10:0:2 $\frac{2}{4}$
31	9:0:4 $\frac{3}{4}$
32	8:0:7 0
33	7:1:0 $\frac{1}{4}$
34	6:1:2 $\frac{2}{4}$
35	5:1:4 $\frac{3}{4}$
36	4:1:7 0
37	3:2:2
38	2:2:6
39	1:3:1
40	0:3:5
41	0:0:0

I will turn this *Subducend* into an *Addend*, such as we use in defective Logarithms, thus:

4. This *Subducend* being 1:0:2 $\frac{1}{1}\frac{1}{2}$ I take its Complement to the next intire (or integral) Barrel above it (*viz.* 2:0:0) and I find its complement to be 0:3:6 $\frac{1}{1}\frac{1}{2}$; therefore my Addend will be $\overline{=}$ $\frac{1}{2}$:3:6 $\frac{1}{1}\frac{1}{2}$.

5. This Addend $\overline{\frac{1}{2}}$:3:6 $\frac{1}{1}\frac{1}{2}$ I write down upon a Card or a piece of stiff Paper, and add it continually to 40:1:3 the whole Content, till I come to 12 Inches wants, and there I find my last Result to be $=$ 27:2:5, which is the Remainer of the Foot Table; whereby I know my Work is true.

6. I come to the second Foot, and by the Rule of Proportion I say, If 12 Inches require 11:3:7:: what shall 1 Inch require? *facit* 0:3:8 $\frac{5}{6}$. Then by the fourth hereof I make my Addend, and find it to be $=$ $\overline{1}$:0:0 $\frac{1}{6}$; and this I add continually to 27:2:5, till I come to 24 Inches wants, and there I find my last Result to be $=$ 15:2:7, which is the Remainer of the Foot Table; whereby I know my Work is true.

7. I come to the third Foot, and by the Rule of Proportion I say; If 12 Inches require 11:1:0:: What shall 1 Inch require? *facit* 0:3:6 $\frac{1}{4}$. Then by the 4th hereof I make my Addend, and find it to be $=$ $\overline{1}$:0:2 $\frac{1}{4}$; and this I add continually to 15:2:7, till

till I come to 36 Inches wants, and there I find my last Result to be = 4 : 1 : 7, which is the Remainer of the Foot Table; whereby I know my Work is true.

8. I come to the lowest Kerf (or Slice) and by the Rule of Proportion I say, If 5 Inches require 4 : 1 : 7 :: What shall 1 Inch require? *facit* 0 : 3 : 5. Then by the 4th hereof I make my Addend, and find it to be = 1 : 0 : 4; and this I continually add to 4 : 1 : 7, till I come to 41 Inches wants, and there I find my last Result to be = 0 : 0 : 0, which proves my Work to be true.

9. Now that you have Inched the whole Tun, you must expunge all your Fractions, for it is not convenient (neither will it be required of you) that Fractions should be annexed to the Table when you write it fair into your Book.

¶ *Of the Drip or Fall of a Tun.*

Something ought here to be spoken of the Fall of a Tun; but because I have handled that in the 7th Chapter of the first Part, I shall say nothing of it here; onely I will tell you what two of our late Writers have said. One of them in Page 87 saith.

"Into the Tun proposed (being empty) pour "Liquor, until the uppermost Part, or Edg of the "Bottom be just cover'd; then with a Gauging-Rod "divided into Inches and decimal Parts) take that "Liquors depth in the deepest Place of it: Lastly, the "one half of that Depth (so taken) is *prope verum* "= the Tuns Fall or Drip; (being always more or "less, according to the Tuns largeness or smallness) "the which being subducted from the Inches, *&c.* of "the Tuns Depth (found by the Rod) leaveth the "Vessels true Depth, to be reckoned on, and made "use of in Inching that Tun. Another of them in Page 70 saith:

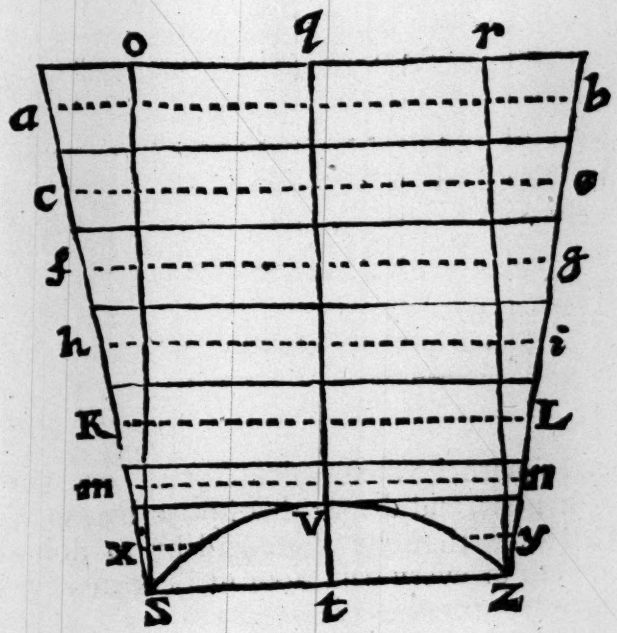
"For the Drip or Fall of the Tun, pour Liquor "there-

"thereinto (it being empty) until the uppermoſt "Part, or Edg of the Bottom be juſt covered; then "take that Liquors depth, in the deepeſt place in In- "ches, *&c.* the one half of which is the Tuns Fall, "which ſubtracted from the Depth, leaves the *true* "*Depth* to be made uſe of in Inching that Tun.

Truly I think theſe two have ſpoken pretty well to the Point, for it ſounds of Equity that half the Fall ſhould be allowed; but in them that will allow nothing of the Fall, I think it to be (not onely a piece of *too much Severity*, but) a piece of Injuſtice.

CHAP. II.

Of the uſual manner of taking mean Diameters in Coppers; and of Gauging the Crown of a Copper.



v, t, = p
s, Z, = d

The

THe usual way is to take the mean Diameters in the middle of every half Foot of the Perpendicular (but if you measure it in the side, you must allow accordingly) So here I take the first mean Diameter a, b, = 103 Inches, the second c, e, = 100, the third f, g, = 94, the fourth h, i, = 90, the fifth K, L, = 86½ Inches; and the next Slice or Kerf being not half a Foot, I draw the Tangent or Touch-line to the *Vertex* of the Crown; (*which I find to be 79 Inches, and this I note down and set it by, till hereafter I shall have occasion to use it*) also I find the thickness of this Slice to be 3 Inches, then in the middle of it, to wit, at 1½ Inch above the *Vertex* of the Crown, I take the mean Diameter m, n, = 81; so have I gotten all the mean Diameters above the *Vertex* of the Crown, and place them in this order in my Book.

Perpend.	Mean diameters.	Mean diameters.	Beer	Ale
6	103	-----	4:3:6	5:2:1
6	100	-----	4:2:5	5:0:7
6	94	-----	4:0:4	4:2:4
6	90	-----	3:3:0	4:0:7
6	86½	-----	3:1:8	3:3:5
3	81	-----	1:2:1	1:2:7
9	77	-----	2:0:5	2:1:5
42			24:2:2	27:2:4

Then I square the mean Diameter of every respective half Foot, and divide that square by 2154, as in the Table for that purpose, and by so doing I get the Content of every half Foot in Beer measure.

Afterward for the Slice next the *Vertex* of the Crown, I square its mean Diameter (to wit, 81) and multiply this Square by its thickness (to wit, 3) and divide

divide this Product 12926, as in the Tablet for that purpose, and so I have the Content of this last Slice next the *Vertex* in beer measure.

¶ *Of Gauging the Crown of a Copper.*

Having, as before, found the half Foot Slices, and also the Slice next the *Vertex* of the Crown; the next thing in order is to find the Content of the Crown it self in Beer measure.

To do this, you must get the length of v, *t*, the Crowns Perpendicular, and also the length of *s*, z, the Diameter of the Base of the Crown; which to attain I let fall the Perpendicular *r*, z, which I find to be 42, and likewise I let fall the Perpendicular *q*, *v*, which I find to be 33: Then I subduct *q*, v, from *r*, z, and the Remainer is *v*, *t*, = 9 Inches, the Crowns Perpendicular: Moreover, I let fall the Perpendicular *o*, *s*, which is = *r*, z, = 42; then I measure the distance *r*, *o*, and find it to be 75, which by the work must be equal to *s*, z, the Diameter of the Base of the Crown.

The Crown of a Copper may be taken in two *Notions*; either as the Frustum of a Sphere intercepted between two Planes parallel, one touching and the other cutting the Sphere; or else as the Frustum of a parabolical *Conoid* intercepted between two Planes parallel, one touching the *Conoid* in the *Vertex*, and the other cutting the *Conoid*.

1. If the Crown shall have equal Curvature at the *Vertex* and towards the Bottom, it is the Frustum of a Sphere.

2. If the Crown shall have more Curvature at the *Vertex* than towards the Bottom, it may be taken for the Frustum of a parabolical *Conoid*.

Then let *s*, v, z, *s*, represent the Crown of a Copper, and if it be taken for the Frustum of a Sphere, Then, if you put *s*, z, = *d*, and v, *t*, = *p*, it holds: $12926) : dd + 1\frac{1}{3}pp : \times \frac{1}{2}p ($ = the Content in Beer measure,

measure, as in the Tablet for that purpose. It may be rendred in words thus:

1. Square the Diameter at the Bottom of the Crown, and to this Square add $1\frac{1}{3}$ of the Square of the Crowns Perpendicular.

2. Multiply this Sum by half the Crowns Perpendicular, then divide this Product by 12926, and you have in the Quotient the Content in Beer measure, as in the Tablet for that purpose.

Example. Here in this Example *d* is equal to 75 Inches, and *p* is equal to 9 Inches, and by the Rule prescribed the Content will be found thus. See the Work.

$75 = d$	$9 = p$
$75 = d$	$9 = p$
375	$81 = pp$
525	$27 = \frac{1}{3} pp$
$5625 = dd$	$108 = 1\frac{1}{3} pp$
$108 = 1\frac{1}{3} pp$	
5733	
$4\frac{1}{2} = \frac{1}{2} p$	
22932	
2866	
25798	B. F. G.
12926	(1 : 3 : $8\frac{1}{4}$ the Crowns Content
12872	in Beer measure.

Having found the Crowns Content, it ought to be taken out of the lowest Slice (which is bounded by the Touch-line (79 found before) and the Diameter of the Crowns Base (*viz.* 75) which Slice is gotten thus:

Square the Arithmetical mean (*x*, y,) between both the Boundaries (for you can by no means take a mean Diameter here actually) and multiply this Square by the thickness or depth of the Slice (which

is

is always equal to the Crowns height) and then divide by 12926, and you have the Content of this Slice in Beer measure, as in the Tablet for that purpose.

Example.

79 = The Touch-line.
75 = The Crowns Diameter.

154 = Their Sum.
77 = Arithmetical mean.
77 = Arithmetical mean.

539
539

5929 = Arithmetical mean squared.
9 = Crowns height.

B. F. G.

53361 (4 : 0 : 4 $\frac{1}{4}$ The Content of the lowest
51704 Slice in Beer measure

1657

Now having found the Content of the lowest Slice, from this I subduct the Content of the Crown, and the Remainer is 2 : 0 : 5.

But if the Crown of a Copper shall be taken for a Frustum of a parabolical *Conoid*; then the Rule holds thus: 12926) dd × $\frac{1}{2}$ p (= The Content in Beer measure.

Example.

75 = d
75 = d

375
525

5625 = dd
4 $\frac{1}{2}$ = $\frac{1}{2}$ p

22500
2812

B. F. G.

25312 (1 : 3 : 7 $\frac{1}{2}$ The Crowns Content in Beer
12926 measure.

12386

CHAP. III.

Of the Explanation of certain Tablets of ready practice for several uses in square Tuns, or oblong Tuns, and Coolers; round Tuns or Elliptical Tuns: also for Coppers and Mash-Tuns.

The Divisor	B.	2 : F :	G.	0 : F :	G.
10152	1	5076	0	0000	0
20304	2	5358	1	0282	1
30456	3	5640	2	0564	2
40608	4	5922	3	0846	3
50760	5	6204	4	1128	4
60912	6	6486	5	1410	5
71064	7	6768	6	1692	6
81216	8	7050	7	1974	7
91368	9	7332	8	2256	8
101520	10				
111672	11	3 : F :	G	1 : F :	G
121824	12	7614	0	2538	0
131976	13	7896	1	2820	1
142128	14	8178	2	3102	2
152280	15	8460	3	3384	3
162432	16	8742	4	3666	4
172584	17	9024	5	3948	5
182736	18	9306	6	4230	6
192888	19	9588	7	4512	7
203040	20	9870	8	4794	8

This is

THis Tablet serves for square Tuns and oblong Tuns, their Dimensions being taken in Inches, and may be of use for Coolers or Backs, if you please.

If you put A = the Area of the Bottom or the Top, for it is here supposed that every Tun and every Cooler is a *Prism*; (that is, hath two Bases equal, parallel, alike, and alike situate, and in the *Peripetasma* a right Line may be every where applied from Base to Base) and P = the perpendicular Height or Depth of the Tun or Cooler: Then it holds,

10152) AP (= B : F : G. Beer measure.

Example 1. Let it be required to find the Content of such a square Tun as in Page the 6th. whose Area at the Top or Bottom is = 11664, and let the perpendicular Height or Depth be = 31 Inches.

```
      108
      108
     ----
      864
     108
    -----
    11664 = A
       31 = P
    -----
    11664
   34992      B. F. G.
   ------
   361584 (35 : 2 : 5>
   30456
   -----
    57024
    50760
    -----
     6464
```

Here you must *Note*, that when I have gotten my 35 Barrels, my Remainer being 6464, I seek it in the Tablet, and because I find 6486 to be the nearest to it,

I take that, and it ſhews me by inſpection 2 F. 5 G. a little too much.

Example 2. Let it be required to find the Content of ſuch an oblong Tun as in Page 7, whoſe Area at the Top or Bottom is = 9408, and let the perpendicular Height or Depth be = 42 ½ Inches.

```
      112
       84
     ----
      448
     896
     ----
     9408 = A
       42 ½ = P
     ------
     18816
    37632
      4604   B. F. G.
    ------
    399840 (39 : 1 : 5 >
    30456
    ------
     95280
     91368
    ------
      3912
```

Here my Remainer being 3912, I find by inſpection that 1 F. 5 G. will paſs.

Example 3. Let it be propoſed to find the Content of ſuch a Triangulate Tun as is in Page 9, whoſe Area at the Top or Bottom is = 5672, and let the perpendicular Height or Depth be = 36 Inches

```
      5672 = A
        36 = P
     ------
     34032
    17016   B. F. G.
    ------
    204192 ( 20 : 0 : 4 <
    203040
    ------
      1152
```

Here my Remainer being 1152, I find by Inſpection that 0 F. 4 G. will paſs.

The

The Divisor	B.	2. F.	G.	0. F.	G.
0846	1	423	0	000	0
1692	2	446	1	024	1
2538	3	470	2	047	2
3384	4	494	3	070	3
4230	5	517	4	094	4
5076	6	541	5	117	5
5922	7	564	6	141	6
6768	8	588	7	164	7
7614	9	611	8	188	8
8460	10				
9306	11	3. F.	G.	1. F.	G.
10152	12	634	0	211	0
10998	13	658	1	235	1
11844	14	681	2	258	2
12690	15	705	3	282	3
13536	16	728	4	305	4
14382	17	752	5	329	5
15228	18	775	6	352	6
16074	19	799	7	376	7
16920	20	822	8	400	8

This Tablet serves for Square Tuns, and Oblong Tuns, their Dimensions being taken in Inches; and the Perpendicular being always 12 Inches, or one Foot.

The Rule.

Multiply together the cross Diameters (or Conjugates) in the middle of every Foot, and divide that Product by 846, the Divisor in this Tablet; and the whole Number of the Quotient is the Beer Bar-

rels, and the Remainer will abutt you to the Firkins and Gallons by inspection.

Example 1. Let it be proposed to find the Content of the first Foot of that square Tun in Page 89.

```
      112
      112
     ----
      224
     112
    112      B. F. G.
   ------
   12544 (14 : 3 : 3 >
   11844
   -----
     700
```

Example 2. Let it be required to find the Content of the first Foot of that oblong Tun in Page 90.

```
      112
      122
     ----
      224
     224
    112      B. F. G.
   ------
   13664 (16 : 0 : 5 <
   13536
   -----
     128
```

The

The Divis.	B.	2 : F : G		0 : F :	G
1692	1	0846	0	000	0
3384	2	0893	1	047	1
5076	3	0940	2	094	2
6768	4	0987	3	141	3
8460	5	1034	4	188	4
10152	6	1081	5	235	5
11844	7	1128	6	282	6
13536	8	1175	7	329	7
15228	9	1222	8	3768	
16920	10				
18612	11	3 : F :	G	1 : F :	G
20304	12	1269	0	432	0
21996	13	1316	2	470	1
33688	14	1363	1	517	2
25380	15	1410	3	564	3
27072	16	1457	4	611	4
28764	17	1504	5	658	5
30456	18	1551	6	705	6
32148	19	1598	7	752	7
33840	20	1645	8	799	8

This Tablet ſerves for ſquare Tuns and oblong Tuns, their Dimenſions being taken in Inches, and the Perpendicular being always 6 Inches or $\frac{1}{2}$ Foot.

The Rule.

Multiply together the croſs Diameters (or Conjugates) in the middle of every half Foot, and divide that Product by 1692, the Diviſor in this Tablet; and the whole Number of the Quotient is the Beer

Barrels, and the Remainer will abutt you to the Firkins and Gallons by Inspection.

Example 1. Let it be proposed that there is a square Tun whose cross Diameters were taken in the middle of every half Foot, and the cross Diameters in the middle of one of the half Feet were 117 and 117: And let it be required to find the Content of that half Foot.

```
   117
   117
  ----
   819
  117
  117        B. F. G.
  -----
  13689 ( 8 : 0 : 3 <
  13536
  -----
   153
```

Example 2. Let it be proposed that there is an oblong Tun, whose cross Diameters were taken in the middle of every half Foot, and the cross diameters in the middle of one of those half Feet were 117 and 123: Now let it be required to find the Content of that half Foot.

```
   117
   123
  ----
   351
  2 4
  117        B. F. G.
  -----
  14391 ( 8 : 2 : 0 <
  13536
  -----
   855
```

The

The Diviſor	Q.
14528	1
29056	2
43584	3
58112	4
72640	5
87168	6
101696	7
116224	8
130752	9
145280	10
159808	11
174336	12
188864	13
203392	14
217920	15
232448	16
246976	17
261504	18
276032	19
290560	20

2 : ſt :	pk
7264	0
7718	1
8172	2
8626	3
9080	4
9534	5
9988	6
10443	7

0 : ſt :	pk
0000	0
0454	1
0908	2
1362	3
1816	4
2270	5
2724	6
3178	7

3 : ft :	pk
10896	0
11350	1
11804	2
12258	3
12712	4
13166	5
13620	6
14074	7

1 : ſt :	pk
3636	0
4086	1
4540	2
4994	3
5448	4
5902	5
6356	6
6810	7

This Tablet ſerves for ſquare Maſh-Tuns and oblong Maſh-Tuns, their Dimenſions being taken in Inches; but here we will conſider them in two Notions, that is, either as *Priſms*, or elſe as Fruſtums of *Pyramids*.

1. If your Maſh-Tun be a *Priſm*, get the Area of its Bottom, which how to do (when it ſhall be a Square, Oblong, Triangulate) hath been taught before in the Firſt Part: And having the Area of its Bottom, that Number you muſt keep in ſtore for your

your uſe, for it is an auxiliary Number, and you wait for nothing but a Perpendicular to multiply with it, and to divide that Product (or Fact) by 14528, the Diviſor in this Tablet, and the whole Number of the Quotient is the Quarters of Mault wetted, and the Remainer abutts you to the Strikes and Pecks by inſpection.

Example 1. Let it be propoſed that there is a ſquare Maſh-Tun whoſe croſs Diameters (or Conjugates) are 131 and 131, and the Depth of the Goods I find to be 17 Inches: Now let it be required to find the quantity of Mault wetted this Brewing.

131
131
131
393
131
17161 = the Area of the Bot...
17 = The Perpendicular.
120127
17161 Q. ſt. p. k.
291737 (20 : 0 : 3
290260
1177

Example 2. Let it be propoſed that there is an oblong Maſh-Tun, whoſe croſs Diameters (or Conjugates) are 131 and 127, and the depth of the Goods I find to be 16 Inches: Now let it be required to find the quantity of Mault wetted this Brewing.

131

131
127
917
262
131
16637 = The Area of the Bottom.
16 = The Perpendicular.
99822
16637
266192 Q. ſt. pk. (18 : 1 : 2 <
261504
4688

2. If your Maſh-Tun be a Fruſtum of a *Pyramid*, you muſt then get the Area in the middle of the uſual depth of the Goods (when the Brewer hath drawn his Length) and this Area you may account for an auxiliary Number (as before) and the Tun is reduced as it were to a *Priſm*; which being done, all things hold as they did before.

If you ask me, and why a Diviſor of 14528? I'le tell you: 'In the Month of *October*, 1666. when I 'was a Gauger in *Middleſex*, Mr. *Porter* and I made 'an Experiment of ſundry ſorts of Mault, and we 'found that although 272 $\frac{1}{4}$ cubical Inches are equal 'to the Corn-Gallon, yet an indifferent ſort of Mault, 'and of indifferent grinding, when three Worts ſhall 'have paſſed through it, it is more compact by a-'bout a ſixth part; therefore he and I did then re-'ſolve upon the Diviſors for ſquare Maſh-Tuns to be '227, and for the round 288.

This being premiſed, it muſt needs follow that 14528 muſt be the Diviſor for Quarters of Mault in ſquare Maſh-Tuns, for $227 \times 64 = 14528$.

I thing it needleſs to make a Tablet for round or Elliptical Maſh-Tuns, for if you get the Area of the Circle,

Circle, or of the Ellipsis (or in more general terms, the Area of any Base whatsoever) this Tablet holds to all Intents and Purposes; yet howsoever, because you may desire a Tablet for round Mash-Tuns, I'le put you one in, in its due place amongst the round Tablets.

The Divisor	B.	2 : F :	G	0 : F :	G
12926	1	6462	0	0000	0
25852	2	6821	1	0359	1
38778	3	7180	2	0718	2
51704	4	7539	3	1077	3
64630	5	7898	4	1436	4
77556	6	8257	5	1795	5
90482	7	8616	6	2154	6
103408	8	8975	7	2513	7
116334	9	9334	8	2872	8
129260	10				
142186	11	3 : F :	G	1 1 : F :	G
155112	12	09693	0	3231	0
168038	13	10052	1	3590	1
180964	14	10411	2	3949	2
193890	15	10770	3	4308	3
206816	16	11129	4	4667	4
219742	17	11488	5	5026	5
232668	18	11847	6	5385	6
245594	19	12206	7	5744	7
258520	20	12565	8	6103	8

This Tablet servs for Round Tuns and Elliptical Tuns, their Dimensions being taken in Inches; supposing every such Tun to be a *Prism*, that is, hath two Bases

Baſes equal, parallel, alike, and alike ſituate, and in the *Peripetaſma* a right Line may be every where applied from Baſe to Baſe.

For round Tuns, if you put D = the Diameter, and P = the Perpendicular Height or Depth: Then it holds:

12926) DD P (= B. F. G. Beer meaſure.

For Elliptical Tuns, if you put D = the greater Conjugate, and d = the leſſer Conjugate, and P = the perpendicular Height or Depth: Then it holds:

12926) D d P (= B. F. G. Beer meaſure.

Example 1. Let there be propoſed that there is a round Tun whoſe Diameter is = 114, and its perpendicular Height or Depth = 37 Inches.

114 = D
114 = D
456
114
114
12996
37 = P
90972
38988 B. F. G.
480852 (37 : 0 : 7 ⊲
38778 ·
93072
90482
2590

Example 2. Let it be propoſed that there is an Elliptical Tun, whoſe greater Conjugate is 114, and the leſſer Conjugate is = 98, and the perpendicular Height or Depth is = 37 Inches.

114 = D
98 = d
912
1026
11172
37 = P
78204
33516 B. F. G.
413364 (31 : 3 : 8 <
38778
25584
12926
12658

Furthermore, this Tablet ſerves for the Fruſtum of a *Cone*, if you put D = the Diameter of the greater Baſe, d = the Diameter of the leſſer Baſe, (making S = D + d) and P = the perpendicular Height or Depth; then it holds:

12926) : S S − D d : × $\frac{1}{3}$ P (= B. F. G. Beer meaſure.

Example 3. Let it be propoſed that there is a Tun (agreed upon to be) the Fruſtum of a *Cone*; whoſe Diameter of the greater Baſe is = 124.21, the Diameter of the leſſer Baſe is = 119.24, and the perpendicular Height or Depth is = 46.5 Inches.

119.24

119.24 = d
124.21 = D
243.45 = S
243.45 = S
121725
97380
73035
97380
48690
59267.9025 = S S
14810.8004 = Dd
44457.1021
15.5 = $\frac{1}{3}$ P
222285 5105
222285 5105
44457 1021 B. F. G.
689085.08255 (53 : 1 : 2 ◁ not 54 : 1 : 3.
64630
42785
38778
4007

119.24 = d
124.21 = D
11924
23848
47696
23848
11924
14810.8004 = Dd

Moreover, this Tablet ſervs for the Crowns of Brewers Coppers, if you put d = the Diameter of the Baſe of the Crown, and p = the perpendicular Height of the Crown; then it holds:

12926) : dd + 1 $\frac{1}{3}$ pp : × $\frac{1}{2}$ p (= B. F. G. Beer meaſure.

Example 4. Let it be propoſed that there is a Coppers Crown whoſe Diameter at the Baſe is = 108, and the perpendicular height of this Crown is = 15 Inches.

108 = d

108 = d
108 = d
864
108
11664 = dd
+ 300 = 1 $\frac{1}{3}$ pp
11964
7 $\frac{1}{2}$ = $\frac{1}{2}$ p
83748
5982 B. F. G.
89730 (6 : 3 : 7
77556
12174

15 = p
15 = p
75
15
225 = pp
75 = $\frac{1}{3}$ pp
300 = 1 $\frac{1}{3}$ pp

Example 5. Let it be propoſed that there is a Coppers Crown whoſe Diameter at the Baſe is = 92, and the Perpendicular Height of this Crown is = 12 Inches.

92 = d
92 = d
184
828
8464 = dd
+ 192 = 1 $\frac{1}{3}$ pp
8656
6 = $\frac{1}{2}$ p
B. F. G.
51936 (4 : 0 : 1
51704
232

12 = p
12 = p
24
12
144 = pp
48 = $\frac{1}{3}$ pp
192 = 1 $\frac{1}{3}$ pp

The

The Divis.	B.	2 : F : G		0 : F	G
1077	1	540	0	000	0
2154	2	570	1	030	1
3231	3	600	2	060	2
4308	4	630		090	3
5385	5	660	4	120	4
6462	6	690	5	150	5
7539	7	720	6	180	6
8616	8	750	7	210	7
9693	9	780	8	240	8
10770	10				
11847	11	3 : F :	G	1 : F	G
12924	12	810	0	270	0
14001	13	840	1	300	1
15078	14	870	2	330	2
16155	15	900	3	360	3
17232	16	930	4	390	4
18309	17	960	5	420	5
19386	18	990	6	450	6
20463	19	1020	7	486	7
21540	20	1050	8	510	8

This Tablet ſerves for Round Tuns and Elliptical Tuns, their Dimenſions being taken in Inches; and the Perpendicular being always 12 Inches, or 1 Foot.

The Rule.

Multiply together the (croſs Diameters or) Conjugates in the middle of every Foot, and divide that Product by 1077, the Diviſor in this Tablet; and the whole Number of the Quotient is the Beer Barrels,

and

and the Remainer will abutt to the Firkins and Gallons by Inſpection.

Example 1. Let it be propoſed to find the Content to the firſt Foot of that round Tun in Page 91.

112
112
224
112
112 B. F. G.
12544 (11:2:5 <
11847
697

Example 2. Let it be required to find the Content of the firſt Foot of that Elliptical Tun in Page 91.

122
112
244
122
122 B. F. G.
13664 (12:2:7 >
12924
740

Example 3. Let there be propoſed an Elliptical Tun, whoſe (croſs Diameters or) Conjugates in the ſecond Foot, are 115 and 123; To find the Content of this ſecond Foot in Beer meaſure.

115
123
345
230
115 B. F. G.
14145 (13:0:5 >
14001
144

The

The Divis.	B.	2 : F :	G	0 : F :	G
2154	1	1080	0	000	0
4308	2	1140	1	060	1
6462	3	1200	2	120	2
8616	4	1260	3	180	3
10770	5	1320	4	240	4
12924	6	1380	5	300	5
15078	7	1440	6	360	6
17232	8	1500	7	420	7
19386	9	1560	8	480	8
21540	10				
23694	11	3 : F :	G	1 : F :	G
25848	12	1620	0	540	0
28002	13	1680	1	600	1
30156	14	1740	2	660	2
32310	15	1800	3	720	3
34464	16	1860	4	780	4
36618	17	1920	5	840	5
38772	18	1980	6	900	6
40926	19	2040	7	960	7
23080	20	2100	8	1020	8

This Tablet ſerves for Coppers, Round Tuns and Elliptical Tuns, their Dimenſions being taken in Inches; and the Perpendicular being alwa y s 6 Inches or $\frac{1}{2}$ Foot.

The Rule.

Multiply together the (croſs Diameters or) Conjugates in the middle of every half Foot, and divide that Product by 2154, the Diviſor in this Tablet; and the

the whole Number of the Quotient is the Beer Barrels, and the Remainer will abutt you to the Firkins and Gallons, by inspection.

Example 1. Let it be proposed that there is a Round Tun whose Conjugates were taken in the middle of every half Foot, and the Conjugates in the middle of one of those half Feet were 117 and 117: And let it be required to find the Content of that half Foot.

```
   117
   117
  ----
   819
  117
 117      B. F. G.
 ------
 13689 (6:1:4>
 12924
 -----
   765
```

Example 2. Let it be proposed that there is an Elliptical Tun whose Conjugates were taken in the middle of every Foot, and the Conjugates in the middle of one of those half Feet were 117 and 123: Now let it be required to find the Content of that half Foot.

```
   123
   117
  ----
   861
  123
 123      B. F. G.
 ------
 14391 (6:2:6<
 12924
 -----
  1467
```

The

The Divisor	Q.	2 : ſt :	pk	0 : ſt :	pk
18432	1	09216	0	0000	0
36864	2	09792	1	0576	1
05296	3	10368	2	1152	2
73728	4	10944	3	1728	3
92160	5	11520	4	2304	4
110592	6	12096	5	2880	5
129024	7	12672	6	3456	6
147456	8	13248	7	4032	7
165888	9				
184320	10	3 : ſt :	pk	1 : ſt :	pk
202752	11	13824	0	4608	0
221184	12	14400	1	5184	1
239616	1	14976	2	5760	2
258048	14	15552	3	6336	3
276480	15	16128	4	6912	4
294912	16	16704	5	7488	5
313344	17	17280	6	8064	6
331776	18	17856	7	8640	7
850208	19				
368640	20				

This Tablet ſerves for round Maſh-Tuns, and Elliptical Maſh-Tuns, their Dimenſions being taken in Inches, and here (as before in Square ones) we muſt conſider them as *Priſms*, or elſe as Fruſtums of *Pyramids*.

1. If your Maſh-Tun be a *Priſm*, get the Fact of its Conjugates, (whether it ſhall be a Circle or an Ellipſis:) And having this Fact, that Number you muſt keep in ſtore for your uſe, for it is an auxiliary Number, and you wait for nothing but a Perpendicular

cular to multiply with it, and to divide that Product by 18432, the Divisor of this Tablet; and the whole Number of the Quotient is the Quarters of Maul: wetted, and the Remainer abutts you to the Strikes and Pecks by inspection.

Example 1. Let it be proposed that there is a round Mash-Tun, whose (cross Diameters or) Conjugates are 131 and 131, and the depth of the Goods I find to be 17 Inches: Now let it be required to find the quantity of Mault wetted this Brewing.

```
   131
   131
  ----
   131
  393
 131
 -----
 17161 = The Fact of the Conjugates.
    17 = The Perpendicular.
 ------
 120127
 17161    Q. st. pk.
 ------
 291737 (15 : 3 : 2 <
 276480
 ------
  15257
```

Example 2. Let it be proposed that there is an Elliptical Mash-Tun whose (cross Diameters or) Conjugates are 131 and 127, and the depth of the Goods I find to be 16 Inches: Now let it be required to find how much Mault was wetted this Brewing.

127

127
131
127
381
127
16637 = The Fact of the Conjugates.
16 = The Perpendicular.
99822
16637 Q. ft. pk
266192 (14 : 1 : 6 <
258048
8144

Example 3. Let it be propoſed that there is an Elliptical Maſh-Tun, whoſe (croſs Diameters or) Conjugates are 52 and 43, and the depth of the Goods I find to be 7 Inches: Now let it be required to find how much Mault was wetted this Brewing.

52
43
156
208
2236 = The Fact of the Conjugates.
7 = The Perpendicular.
Q. ft. p k.
15652 (0 : 3 : 3 <

2. If your Maſh-Tun be a Fruſtum of a *Pyramid*, you muſt then get the Fact of the Conjugates in the middle of the uſual depth of the Goods, and this Fact you may account for an auxiliary Number (as before) and the Tun is hereby reduced as it were to a *Priſm*; which being done all things hold as they did before.

2:F:	0:F:	G	divi:	B	2:F:	G	0:F:	G
.50	.00	0	12	1	6.00	0	0.00	0
.53	.03	1	24	2	6.33	1	0.33	1
.56	.06	2	36	3	6.66	2	0.66	2
.58	.08	3	48	4	7.00	3	1.00	3
.61	.11	4	60	5	7.33	4	1.33	4
.64	.14	5	72	6	7.66	5	1.66	5
.67	.17	6	84	7	8.00	6	2.00	6
.69	.19	7	96	8	8 33	7	2.33	7
.72	.22	8	108	9	8.66	8	2.66	8
3:F:	1:F:	G	120	10	3:F:	G	1:F:	G
.75	.25	0	132	11	9.00	0	3.00	0
.78	.28	1	144	12	9 33	1	3.33	1
.81	.31	2	156	13	9.66	2	3.66	2
.83	.33	3	168	14	10.00	3	4.00	3
.86	.36	4	180	15	10 33	4	4.33	4
.89	.39	5	192	16	10 66	5	4.66	5
.92	.42	6	204	17	11.00	6	5.00	6
.94	.44	7	216	18	11.33	7	5.33	7
.97	.47	8	228	19	11.66	8	5.66	8
			240	20				

These two Tablets are jointly to be uſed, for finding the Content (Beer meaſure) in any Tun which is already caſt up to every Foot; the whole Perpendicular being partly wet and partly dry.

The firſt Tablet ſerves to reduce Firkins and Gallons (Beer meaſure) to two places of Decimals of Barrels; *& contra*: To reduce two places of Decimals of Barrels; to Firkins and Gallons.

The ſecond Tablet ſerves to find the Content of any part of a Foot-ſlice, being partly wet and partly dry; that

that whole Slice being given in Barrels decimalized to two places, and the wet part of the Perpendicular (in that Foot-slice) being known.

The Rule.

1. If the Foot-slice partly wet and partly dry, be terminated in Firkins and Gallons; Let it be reduced to Barrels decimalized to two places, by the first Tablet: which may be done by inspection.

2. Multiply the Barrels so decimalized to two places, by the wet part of the Perpendicular, in that Foot-slice partly wet and partly dry, and divide that Product by 12, the Divisor in the second Tablet; and the whole Number of the Quotient is the Beer Barrels, and the Remainer will abutt you to the Firkins and Gallons in the second Tablet by inspection.

3. Having thus found the Content of the wet part of that Foot-slice which was in contest; you must to it add all the wet below it, and so you have the Content of all the Liquor that is in the Tun. Which was to be done.

Example 1. Let the Elliptical Tun in Page 91 be proposed, and let it be required to find the Content at 19 Inches wants.

Here because there is 19 Inches dry, the second Foot is that which is in contest, and 'tis also evident that the Perpendicular in this Foot is 7 dry and 5 wet and the Work will stand thus.

B. F. G.
11 : 3 : 7 = 11.94

5 B F. G.

59.70 (4 : 3 : 8
48 15 : 2 : 7
12 20 : 2 : 6 = the Content required.

Example 2. Let the foresaid Tun be proposed again,

gain, and let it be required to find the Content at 28 Inches wants.

Here, becauſe there is 28 Inches dry, the third Foot is that which is in diſpute, and 'tis alſo evident that the Perpendicular in this Foot is 4 dry and 8 wet; and the work ſtands thus.

B. F. G.
11: 1 :0 = 11.25
8 B. F. G.
90.00 (7 : 2 : 0
84 4 : 1 : 7
6 11 : 3 : 7 = The Content required.

Example 3. Let the oblong Tun in Page 90 be propoſed, and let it be required to find the Content at 17 Inches wants.

Here, becauſe there is 17 Inches dry, the ſecond Foot is that which is in diſpute, and 'tis alſo apparent that the Perpendicular in this Foot is 5 dry and 7 wet; and the work ſtands thus.

B. F. G.
15: 0 : 8 = 15.22
7 B. F. G.
106.54 (8 : 3 : 5
96 20 : 0 : 1
10 28 : 3 : 6 = the Content required

Example 4. Let the round Tun in Page 91 be propoſed, and let it be required to find the Content at 7 $\frac{1}{2}$ wants.

Here, becauſe there is 7 $\frac{1}{2}$ Inches dry, the firſt Foot is that which is in diſpute, and it is manifeſt that the Perpendicular in this Foot is 7 $\frac{1}{2}$ dry and 4 $\frac{1}{2}$ wet; and the work is thus.

B. F. G.

B. F. G.
11 : 2 : 5 = 11.64
4 ½
4656
582
52.38 (4 : 1 : 4
48 25 : 1 : 8
4 29 : 3 : 3 = the Content requir'd

If any man ſhall think it better to uſe the old way by the Rule of Practice, he muſt then underſtand the *Aliquot* parts of 12; to wit, thoſe Diviſors which will evenly divide 12, which are here ſet down in this Tablet.

6	½
4	⅓
3	¼
2	⅙
1½	⅛
1⅓	1/9
1¼	1/10
1	1½

This being premiſed, the Operation of the laſt four Examples will ſtand thus.

Example 1.

11 : 3 : 7 = the Foot in conteſt,
5 Inches wet.
3 : 3 : 8 ⅓ = 4 Inches wet.
0 : 3 : 8 ¼ = 1 Inch wet.
15 : 2 : 7 = the wet below.
20 : 2 : 6 = the Content required.

Example 2.

11 : 1 : 0 = The Foot in conteſt; 8 Inches wet.
3 : 3 : 0 = 4 Inches wet,
3 : 3 : 0 = 4 Inches wet,
4 : 1 : 7 = the wet below,
11 : 3 : 7 = the Content required.

Example 3.

15 : 0 : 8 = The Foot in Conteſt; 7 Inches wet

5 : 0 : 2 $\frac{2}{3}$ = 4 Inches wet,
3 : 3 : 2 = 3 Inches wet,
20 : 0 : 1 = the wet below,

28 : 3 : 5 $\frac{2}{3}$ = The Content required.

Example 4.

11 : 2 : 5 = the Foot in conteſt: 4 $\frac{1}{2}$ Inches wet.

2 : 3 : 5 $\frac{3}{4}$ = 3 Inches wet,
1 : 1 : 7 $\frac{1}{2}$ = 1 $\frac{1}{2}$ Inches wet,
25 : 1 : 8 = the wet below.

29 : 3 : 3 = the Content required.

This Tablet of the Aliquot parts of 12, will ſerve as well for Ale meaſure as Beer meaſure; if you remember that 8 Gallons is equal to a Firkin of Ale.

Example 5. Let the Elliptical Tun in Page 91 be propoſed, and let it be required to find the Content (Ale meaſure) at 19 Inches wants.

13 : 1 : 6 = The Foot in Conteſt; 5 Inches wet,

4 : 1 : 7 $\frac{1}{3}$ = 4 Inches wet,
1 : 0 : 3 $\frac{3}{4}$ = 1 Inch wet,
17 : 2 : 5 = the wet below,

23 : 1 : 0 = The Content required.

CHAP. IV.

Of the uſual manner of giving in the Charge upon a common Brewer, for a Guile of Beer or Ale.

Daniel Dawſon of *Dover*, the Dimenſions and Contents of his Brewing Veſſels.

Perpen-

	Per-pen.	Mean diam.	Mean diam.	B	A
1. S. T.	12	123	- - -	17:3:5	20:0:4
	12	121	- - -	17:1:2	19:1:7
	12	118	- - -	16:1:7	18:2:0
	06	115	- - -	07:3:2	08:3:1
	42			59:1:7	66:3:4
2 ob. T.	12	104	109	13:1:5	15:0:2
	12	102	106	12:3:1	14:1:4
	12	101	104	12:1:6	13:3:7
	12	099	102	11:3:7	13:1:6
	05	097	100	04:3:1	05:1:4
	53			55:1:2	62:0:7
3 R. T.	12	102	- - -	09:2:6	10:3:4
	12	100	- - -	09:1:1	10:1:6
	12	097	- - -	08:2:6	09:3:2
	12	095	- - -	08:1:5	09:1:6
	07	094	- - -	04:3:1	05:1:4
	55			40:3:3	45:3:6
4 El. T.	12	107	110	10:3:6	12:1:1
	12	105	108	10:2:1	11:3:3
	12	101	105	09:3:4	11:0:3
	08	098	102	06:0:7	06:3:7
	44			37:2:0	42:0:6

	Per-pen.	Mean diam.	Mean diam.	B	A
1 Back	01	138	104	1:1:6	1:2:3
2 Back	01	205	181	3:2:5	4:0:3
3 Back	01	117	117	1:1:3	1:2:0
4 Back	01	182	195	3:2:0	3:3:6
5 Back	01	141	092	1:1:1	1:1:6
Copper	06	107	106	05:1:0	05:3:5
	06	108	107	05:1:4	06:0:1
	06	109	108	05:1:8	06:0:5
	06	108	108	05:1:6	06:0:3
	06	106	105	05:0:6	05:3:2
	06	104	103	04:3:7	05:2:2
	06	102	100	04:2:8	05:1:2
	02	100	100	01:2:2	01:3:0
	09	098	098	03:1:7	03:3:4
	53			41:1:3	46:2:0
				Q.ft.pk	
R.M.T.	01	142	- - -	1:0:3	

The Dimenſions and Contents of theſe Veſſels being all known, I am capacitated to find the Length of any Guile, at one, two, or three takings.

Example. 1677. *October* 9. *Morn.* 5.

1 B : X : 7 is = 9 : 3 : 6
2 B : X : 2 is = 7 : 1 : 1
3 B : A : 6½ is = 9 : 3 : 0
5 B : VI : 3 is = 3 : 3 : 3
3 R : A : 42 is = 10 : 0 : 3
1 S : X : 33 is = 11 : 3 : 6
4 El : X : 24 is = 16 : 0 : 2

And

And ſuppoſe I come at 7 in the Evening the ſame day, for my ſecond taking.

1677. October 9. Even. 7.

3 R : A : 30 is = 19 : 2 : 7
4 El: X : 15 is = 23 : 3 : 8
1 S : X : 21 is = 21 : 2 : 5
2 ob: VI : 49 is = 03 : 3 : 2

This ſecond taking you ſee I have it all down in the Tuns; therefore 'tis paſt all diſpute, for it is a fair Charge: Howbeit if the Brewer had not ſhewed fair in his Tuns, I would have charged him in the Backs; but theſe two takings agree pretty well together, and therefore I'le charge him with this ſecond taking.

B. F. G.

Strong Ale qt 19 : 2 : 7 that is 631 Gallons.
Strong Beer qt 45 : 2 : 4 that is 1642 Gallons.
Six ſh. Beer qt 03 : 3 : 2 that is 137 Gallons.

So much may ſuffice for ſhewing the uſual manner of giving in a Charge upon a common Brewer.

CHAP. V.

Of the Reduction of Barrels of Beer to Gallons & contra, *and Barrels of Ale to Gallons*, & contra.

IN regard that the late Act of Parliament obligeth the Gauger to give in his Charge (upon the common Brewer and Victualler) in Gallons; therefore it behoveth the Gauger to be ready at this kind of Reduction. I know that every ingenious Gauger is apt enough to invent divers ways to himſelf, and he may (nay he will) take that way which likes him beſt; therefore I ſhall onely offer (not impoſe) my way in theſe four Propoſitions.

Prop. 1. A number of Barrels of Beer, Firkins and Gallons, being propoſed; to reduce all to Gallons.

The Rule.

1. Multiply the propoſed Number of Barrels *mentally* by 6, and take half that Product, ſetting it one place nearer to the Left hand.

2. Multiply the Firkins *mentally* by 9, and thereto put the Gallons.

3. Set this Sum orderly under the two former, then add them together, and you have reduced all to Gallons. *Example.*

B. F. G.
116 : 3 : 5 Beer meaſure.

696
348
32

4208 = Gallons.

Prop. 2. A number of Gallons being propoſed, to reduce them to Barrels of Beer, Firkins and Gallons.

The Rule.

1. Divide the propoſed number of Gallons *mentally* by 9, and ſet the Remainer to toward the right Hand with a Colon betwixt.

2. Divide this laſt Quotient *mentally* by 4, and ſet the Remainer toward the right Hand, with a Colon betwixt. So the laſt Quotient is Barrels of Beer, and the two Remainers are Firkins and Gallons.

Example.

4208 = Gallons

467 : 5 G
B 116 : 3 F } Beer meaſure.

Prop. 3. A number of Barrels of Ale, Firkins, and

and Gallons, being proposed: To reduce all to Gallons.

The Rule.

1. Multiply the proposed number of Barrels *mentally* by 2, and add this double to the proposed number of Barrels, setting the Sum one place nearer to the left Hand.

2. Multiply the Firkins *Mentally* by 8, and thereto put the Gallons.

3. Set this Sum orderly under the two former; then add them together and you have reduced all to Gallons. *Example.*

B. F. G.
116: 3 : 5 Ale measure.

232
348
29

3741 = Gallons.

Prop. 4. A number of Gallons being proposed To reduce them to Barrels of Ale, Firkins, and Gallons.

The Rule.

1. Divide the proposed number of Gallons *mentally* by 8, and set the Remainer toward the right hand, with a Colon betwixt.

2. Divide this last Quotient *mentally* by 4, and set the Remainer toward the right Hand, with a Colon betwixt: So the last Quotient is Barrels of Ale, and the two Remainers are Firkins and Gallons.

Example.

3741 = Gallons.

467 : 5 G. } Ale measure.
B. 116 : 3 F. }

Some

Some perhaps may rather desire Tables, whereby to perform any one of these four Propositions by inspection; for whose sakes I have annexed the two following Tables, which are so easie that they need no Explanation.

A Table

A Table for reducing Barrels of Beer to Gallons, *& contra.*

Barrels	Gallons.	Barrels	Gallons.	Barrels	Gallons.	Barrels	Gallons.	Barrels	Gallons.
1	36	26	936	51	1836	76	2736	200	7200
2	72	27	972	52	1872	77	2772	300	10800
3	108	28	1008	53	1908	78	2808	400	14400
4	144	29	1044	54	1944	79	2844	500	18000
5	180	30	1080	55	1980	80	2880	600	21600
6	216	31	1116	56	2016	81	2916	700	25200
7	252	32	1152	57	2052	82	2952	800	28800
8	288	33	1188	58	2088	83	2988	900	32400
9	324	34	1224	59	2124	84	3024	1000	36000
10	360	35	1260	60	2160	85	3060		
11	396	36	1296	61	2196	86	3096		
12	432	37	1332	62	2232	87	3132		
13	468	38	1368	63	2268	88	3168		
14	504	39	1404	64	2304	89	3204		
15	540	40	1440	65	2340	90	3240		
16	576	41	1476	66	2376	91	3276		
17	612	42	1512	67	2412	92	3312		
18	648	43	1548	68	2448	93	3348		
19	684	44	1584	69	2484	94	3384		
20	720	45	1620	70	2520	95	3420		
21	756	46	1656	71	2556	96	3456		
22	792	47	1692	72	2592	97	3492		
23	828	48	1728	73	2628	98	3528		
24	864	49	1764	74	2664	99	3564		
25	900	50	1800	75	2700	100	3600		

A Table for reducing Barrels of Ale to Gallons, & *contra.*

Barrels	Gallons.	Barrels	Gallons.	Barrels	Gallons.	Barrels	Gallons.	Barrels	Gallons.
1	32	27	832	51	1632	76	2432	200	6400
2	64	28	864	52	1664	77	2464	300	9600
3	96	29	896	53	1696	78	2496	400	12800
4	128	30	928	54	1728	79	2528	500	16000
5	160	31	960	55	1760	80	2560	600	19200
6	192	32	992	56	1792	81	2592	700	22400
7	224	33	1024	57	1824	82	2624	800	25600
8	256	34	1056	58	1856	83	2656	900	28800
9	288	35	1088	59	1888	84	2688	1000	32000
10	320	36	1120	60	1920	85	2720		
11	352	37	1152	61	1952	86	2752		
12	384	38	1184	62	1984	87	2784		
13	416	39	1216	63	2016	88	2816		
14	448	40	1248	64	2048	89	2848		
15	480	41	1280	65	2080	90	2880		
16	512	42	1312	66	2112	91	2912		
17	544	43	1344	67	2144	92	2944		
18	576	44	1376	68	2176	93	2976		
19	608	45	1408	69	2208	94	3008		
20	640	46	1440	70	2240	95	3040		
21	672	47	1472	71	2272	96	3072		
22	704	48	1504	72	2304	97	3104		
23	736	49	1536	73	2336	98	3136		
24	768	50	1568	74	2368	99	3168		
25	800		1600	75	2400	100	3200		

CHAP. VI.

Of finding at one Operation the Duty-money *that any number of* Gallons *of ſtrong Beer, ſtrong Ale, or ſmall Beer, will amount to; the reſpective Allowances not being ſtruck off.*

BEfore I enter into this Chapter, it will not be amiſs briefly to explain the property and uſe of a decimal Fraction in Coin.

A decimal Fraction (then) is a Fraction whoſe Denominator is 10, or ſome poteſtate of 10: As $\frac{7}{10}$, $\frac{7}{100}$, $\frac{7}{1000}$, *&c.* But it may be more commodiouſly expreſt without its Denominator, thus: 0.7, 0.07, 0.007: And foraſmuch as one Farthing in our *Engliſh* Coin is $\frac{1}{960}$ of a Pound ſterling; therefore 0.001 is ſomething leſs than a Farthing; and becauſe 2 *s.* is the $\frac{1}{10}$ of 1 *l.* it may be expreſt in Decimals thus 0.1, and for the ſame reaſon 1 *s.* is expreſt thus 0.05, and 6 *d.* thus 0.025, and 3 *d.* thus 0.0125: But I told you before that 1 in the third place from unity was leſs than a Farthing; therefore you need not trouble your ſelf with any figure ſtanding in the fourth place; but it ſhall ſuffice if you value a *decimal Fraction* in Coin but to the third place, provided you get all the whole ſix-pences, *viz.* that the Remainer after valuation may be leſs than 25, and ſuch a Remainer you may diſcreetly value for Farthings. *Now come we to the Contents of this Chapter.*

1. Foraſmuch as the Duty-mony of 36 Gallons of ſtrong Beer is 3 *s.* 3 *d.* that is in Decimals 0.1625 *l.* ſterling; therefore it holds by proportion direct.

As 36 Gallons is to 0.1625 *l.* ſterling :: So is any number of Gallons to the Duty-mony thereof.

And

And becauſe the firſt and ſecond terms are conſtant, you may make to your ſelf a conſtant or ſtanding Factor thus.

36) 0.1625 (= 0.004514 >

2. And in regard that the Duty-mony of 32 Gallons of ſtrong Ale is 3 *s.* 3 *d.* that is in Decimals 0.1625 *l.* ſterling; therefore it holds by proportion direct.

As 32 Gallons is to 0.1625 *l.* ſterling:: So is any number of Gallons to the Duty-mony thereof.

And becauſe the firſt and ſecond terms are conſtant, you may make to your ſelf a conſtant Factor thus.

32) 0.1625 (= 0.005078 <

3. And the Duty-mony of 36 Gallons of ſmall Beer being 9 *d.* that is in Decimals 0.0375 *l.* ſterling; therefore it holds by proportion direct.

As 36 Gallons is to 0.0375 *l.* ſterling:: So is any number of Gallons to the Duty-mony thereof.

And becauſe the firſt and ſecond terms are conſtant, you may make to your ſelf a conſtant Factor thus:

36) 0,0375 (= 0.001042 >

Now having theſe three Factors, you may make to your ſelf three Tablets of them, which will be ready for your uſe.

Tablet

Tablet for ſtr. Beer.	G	Tablet for ſtr. Ale.	G	Tablet for ſm. Beer.	G
.004514	1	.005078	1	.001042	1
.009028	2	.010156	2	.002083	2
.013542	3	.015234	3	.003125	3
.018056	4	.020312	4	.004167	4
.022569	5	.025391	5	.005208	5
.027083	6	.030469	6	.006250	6
.031597	7	.035547	7	.007292	7
.036111	8	.040625	8	.008333	8
.040625	9	.045703	9	.009375	9

Example 1. What is the Duty-mony for 10976 Gallons of ſtrong Beer?

10976
27083
31597
40625
4514

l. s. d.

49.545553 = 49: 10: 10 $\frac{1}{4}$ the Anſwer.

Example 2. What is the Duty-mony for 10976 Gallons of ſtrong Ale?

10976
30469
35547
45703
5078

l. s. d.

55.736239 = 55: 14: 8 $\frac{1}{4}$ the Anſwer.

Example.

Example 3. What is the Duty-mony for 10976 Gallons of ſmall Beer?

10976		
6250		
7292		
9375		
1042	*l.*	*s.* *d.*
11.436670 =	11 :	08 : 8 $\frac{3}{4}$ the Anſwer.

CHAP. VII.

Of finding at one Operation the neat Duty-mony *that any number of Gallons of ſtrong Beer, ſtrong Ale, or ſmall Beer, will amount to; the reſpective Allowances being ſtruck off.*

IN the Chapter beforegoing it is manifeſt that the Duty-mony of 36 Gallons of ſtrong Beer is 0.1625 *l.* ſterling; and becauſe that the Allowance for ſtrong Beer is 23 *pro* 20; therefore it holds that the *neat Duty-mony* for one Gallon of ſtrong Beer is 0.003925 <.

And alſo by the foregoing Chapter it is manifeſt that the Duty-mony of 32 Gallons of ſtrong Ale is 0.1625 *l.* ſterling: and becauſe that the Allowance for ſtrong Ale is 22 *pro* 20, therefore it holds that the *neat Duty-mony* for one Gallon of ſtrong Ale is 0.004616 <.

Moreover, by the foregoing Chapter it is manifeſt that the Duty-mony of 36 Gallons ſmall Beer is 0.0375 *l.* ſterling: And becauſe that the Allowance for ſmall Beer is 23 *pro* 20, therefore it holds that the *neat Duty-mony* for one Gallon of ſmall Beer is 0.000906 >

Now having theſe three Factors you may make to your ſelf three Tablets of them, which will be ready for your uſe.

Tablet

Tablet for ſtr. Beer.	G	Tablet for ſtr. Ale.	G	Tablet for ſm. Beer.	G
.003925	1	.004616	1	.000906	1
.007850	2	.009233	2	.001812	2
.011775	3	.013849	3	.002717	3
.015700	4	.018466	4	.003623	4
.019626	5	.023082	5	.004529	5
.023551	6	.027699	6	.005435	6
.027476	7	.032315	7	.006340	7
.031401	8	.036932	8	.007246	8
.035326	9	.041548	9	.008152	9

Example 1. What is the *neat Duty-mony* for 10976 Gallons of ſtrong Beer?

10976

23551
27476
35326
3925

l. s. d.

43.080911 = 43 : 01 : 07 $\frac{1}{4}$ the Anſwer.

Example 2. What is the *neat Duty-mony* for 10976 Gallons of ſtrong Ale?

10976

27699
32315
41548
4616

l. s. d.

50.665649 = 50 : 13 : 3 $\frac{3}{4}$ the Anſwer.

Example

Example 3. What is the *neat Duty-mony* for 10976 Gallons of ſmall Beer?

10976			
5435			
6340			
8152			
906	*l.*	*s.*	*d.*
9.944035 =	9 :	18 :	10 ½ the Anſwer.

The

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ERRATA.

Page	Line	Errors	Corrected.
18	9	Frustum	Frustums
18	20	A & G the two	A & G = the two
18	22	B & H the two	B & H = the two
18	24	A & B opposite = the two	A & B = the two opposite
18	28	G & H opposite = the two	G & H = the two opposite
23	21	Nbmber	Number
30	12	0.000002:0337	0.000020337
50	26	*Section*	*Parahraph*
54	16	*Example*	*Example* 1.
56	23	*Example* 2.	*Example* 3.
56	23	Ulage	Ullage
57	14	Hlage	Ullage
57	17	Ulage	Ullage
64	7	.0978	.0977
64	7	.9022	.9023
64	10	.1004	.1005
64	10	.9996	.8995
70	19	Chordlines	Parallel Chordlines
90	18	16 : 1 : 5	16 : 0 : 5
90	18	18 : 1 : 6	18 : 0 : 5
90	22	51 : 2 : 4	51 : 1 : 4
90	22	58 : 0 : 2	57 : 3 : 1
91	16	(*viz* 5)	*dele*
91	17	Slice,	Slice (*viz.* 5)
102	15	4604	4704
107	133	636	3632

ERRATA.

Page 26, *Example* 2. Should ſtand thus:

0.003859 = The ſecond aux. Number.
20 ½ = The Herpendicular.

0.077180
1929
1.708000 = The third aux. Number,

1.787109
20 ½ = The Perpendicular.

35742180
893555

36.635735 = The Content at 20 ½ wet.

B. F. G.
That is 36: 2: 5 >.

Page 29, *Example* 2. ſhould ſtand thus:

0.0002278 = The firſt aux. Number.
9 = The Perpendicular.

0.00204732
0.03071020 = The ſecond aux. Number.

0.03275752
9 = The Perpendicular.

0.29481768
1.38196000 = The third aux. Number.

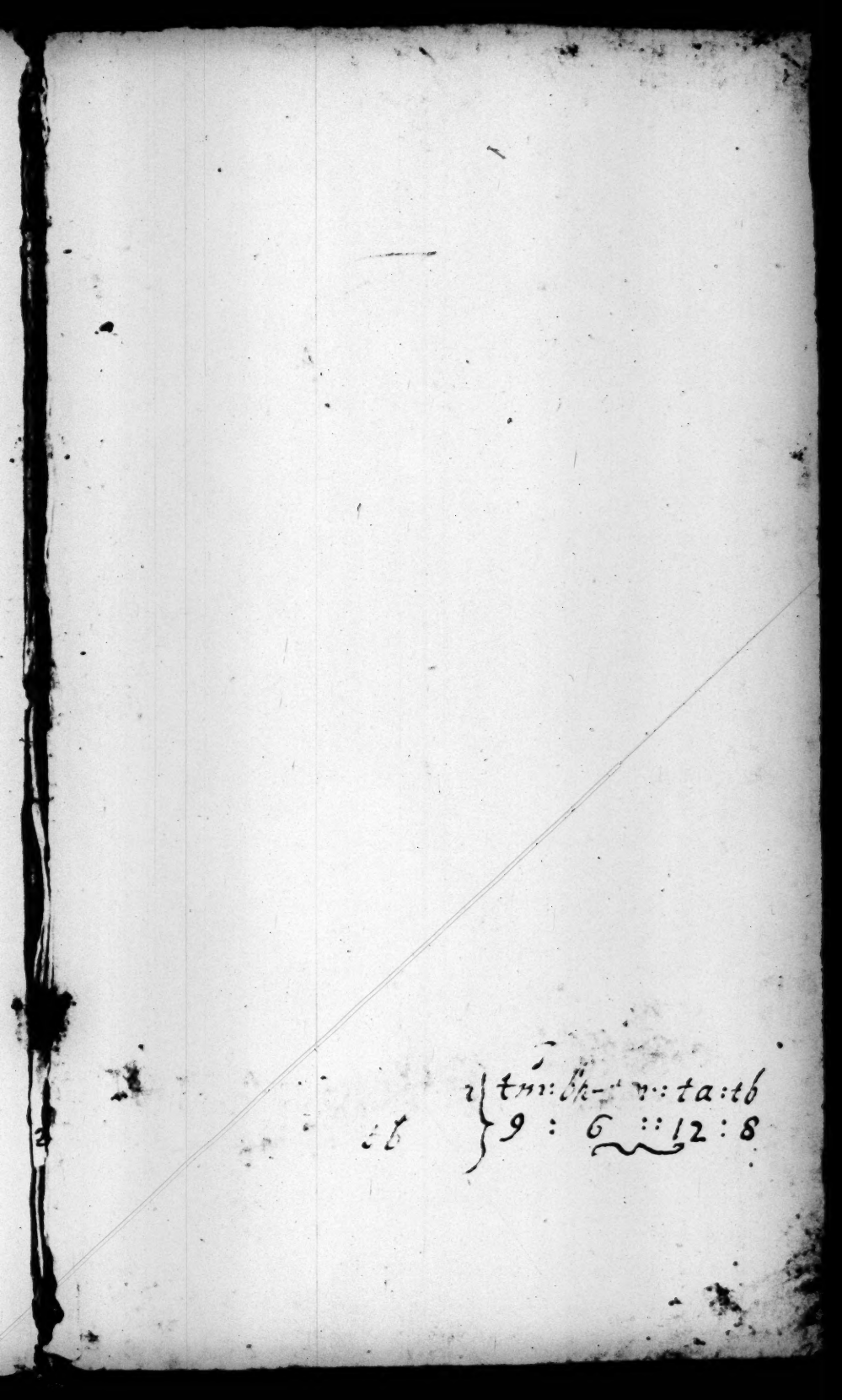
1.67677768
9 = The Perpendicular.

15.09099912 = The Content at 9 Inches wet.

B. F. G.
That is 15: 0: 3 >.

FINIS.

26 MAR 68


tm : bk :: ta : tb
9 : 6 :: 12 : 8